9.36 Let the system be the cart together with all the rain that falls into it.
a) Since all the external forces (gravity, for example) involved are in the vertical direction, the total momentum of the system in the horizontal direction is conserved.
b) Let \( \hat{i} \) be in the direction the cart is moving, and let \( m \) be the mass of the rain that falls into the cart. Before the rain falls into the cart, the velocity of the rain in the \( \hat{i} \) direction is zero. Afterwards, it is the same as the cart’s hence
\[
\vec{p}_{\text{horizontal before}} = \vec{p}_{\text{horizontal after}} \\
(400 \text{ kg})(10.0 \text{ m/s})\hat{i} + m(0 \text{ m/s})\hat{i} = (400 \text{ kg})(8.00 \text{ m/s})\hat{i} + m(8.00 \text{ m/s})\hat{i} \\
\Rightarrow m = 100 \text{ kg}.
\]

9.37
a) This is a stupid collision! It is also completely inelastic, since the cars stick together.
b) Convert the speeds from km/h to m/s:
\[
60.0 \text{ km/h} = (60.0 \text{ km/h}) \left( \frac{10^3 \text{ m}}{\text{km}} \right) \left( \frac{\text{h}}{3600 \text{ s}} \right) = 16.7 \text{ m/s}.
40.0 \text{ km/h} = (40.0 \text{ km/h}) \left( \frac{10^3 \text{ m}}{\text{km}} \right) \left( \frac{\text{h}}{3600 \text{ s}} \right) = 11.1 \text{ m/s}.
\]
The total momentum before the collision is the vector sum of the cars’ individual momenta:
\[
\vec{p}_{\text{before}} = (1.00 \times 10^3 \text{ kg})(16.7 \text{ m/s})\hat{i} + (1.00 \times 10^3 \text{ kg})(11.1 \text{ m/s})\hat{j}
= (16.7 \times 10^3 \text{ kg} \cdot \text{m/s})\hat{i} + (11.1 \times 10^3 \text{ kg} \cdot \text{m/s})\hat{j}.
\]
c) The total momentum after the collision must be the same, since the total momentum is conserved in a collision, so
\[
\vec{p}_{\text{after}} = \vec{p}_{\text{before}} = (16.7 \times 10^3 \text{ kg} \cdot \text{m/s})\hat{i} + (11.1 \times 10^3 \text{ kg} \cdot \text{m/s})\hat{j}.
\]
d) Let \( \vec{v} \) be the velocity of the crumpled mass immediately after the collision. Then, since the total mass of the cars is 2.00 \times 10^3 \text{ kg},
\[
\vec{v} = \frac{1}{2.00 \times 10^3} \vec{p}_{\text{after}} = \frac{1}{2.00 \times 10^3} \left( (16.7 \times 10^3 \text{ kg} \cdot \text{m/s})\hat{i} + (11.1 \times 10^3 \text{ kg} \cdot \text{m/s})\hat{j} \right)
= (8.35 \text{ m/s})\hat{i} + (5.55 \text{ m/s})\hat{j},
\]
and the speed is
\[
v = \sqrt{(8.35 \text{ m/s})^2 + (5.55 \text{ m/s})^2} = 10.0 \text{ m/s}.
\]
e) The change in the kinetic energy is
\[
\Delta \text{KE} = \text{KE}_{\text{total f}} - \text{KE}_{\text{total i}}
= \frac{(2.00 \times 10^3 \text{ kg})(10.0 \text{ m/s})^2}{2} - \left( \frac{(1.00 \times 10^3 \text{ kg})(16.7 \text{ m/s})^2}{2} + \frac{1.00 \times 10^3 \text{ kg})(11.1 \text{ m/s})^2}{2} \right)
= -1.01 \times 10^5 \text{ J}.
\]
Where did all of this energy go? Probably most went into deforming the two car bodies. Some even went into making the load “crash” sound. Notice that about half of the initial kinetic energy went into these processes.

9.38
a) The table does not move since there is no frictional force between the block and the table surface.
b) When the block and bullet system falls off the table it becomes a projectile. We’ll first use the range and height of the flight, along with the equations for motion with a constant acceleration, to find the horizontal speed \( v_{\text{system}} \), at which the system left the table. We’ll then use this together with conservation of momentum to find the speed \( v_{\text{bullet}} \) of the bullet just before impact.

Let \( \mathbf{j} \) point straight up, let \( \mathbf{i} \) point in the direction of the horizontal component of velocity, and choose the origin directly below the point at which the block and bullet leave the table.

\[
\begin{align*}
\text{x direction} & & \text{y direction} \\
\quad v_x(t) = v_{x0} + a_xt = v_{\text{system}}. & & \quad v_y(t) = v_{y0} + a_yt = -gt. \\
\quad x(t) = x_0 + v_{x0}t + \frac{a_x t^2}{2} = v_{\text{system}} t. & & \quad y(t) = y_0 + v_{y0}t + \frac{a_y t^2}{2} = 0.750 \text{ m} - \frac{g t^2}{2}.
\end{align*}
\]

On impact with the floor, the \( y \) coordinate is zero, so we may use this in the \( y(t) \) equation to find the time \( t_{\text{impact}} \) of impact.

\[
0 \text{ m} = 0.750 \text{ m} - \frac{g t_{\text{impact}}^2}{2} \implies t_{\text{impact}} = 0.391 \text{ s}.
\]

The \( x \) coordinate on impact is 2.50 m, so from the \( x(t) \) equation,

\[
2.50 \text{ m} = v_{\text{system}} 0.391 \text{ s} \implies v_{\text{system}} = 6.39 \text{ m/s}.
\]

So, the completely inelastic collision of the bullet with the block of wood results in a composite particle traveling at speed \( v_{\text{system}} = 6.39 \text{ m/s} \).

Now let \( m_{\text{bullet}} \) and \( m_{\text{block}} \) be the masses of the bullet and block. The momentum of the bullet-block system is conserved before and after the collision, so

\[
\mathbf{p}_{\text{total before}} = \mathbf{p}_{\text{total after}} \implies m_{\text{bullet}} v_{\text{bullet}} \mathbf{i} = (m_{\text{bullet}} + m_{\text{block}}) v_{\text{system}} \mathbf{i}
\]

\[
\implies v_{\text{bullet}} = \left( m_{\text{bullet}} / m_{\text{bullet}} + m_{\text{block}} \right) \text{ v}_{\text{system}} = \left( \frac{0.0100 \text{ kg} + 3.00 \text{ kg}}{0.0100 \text{ kg}} \right) 6.39 \text{ m/s} = 1.92 \times 10^3 \text{ m/s}.
\]

9.39 Let \( \mathbf{i} \) point in the direction of motion of the first mass.

a) The total momentum before the first collision is just the momentum of the incident particle, since the remaining four particles are at rest.

\[
\mathbf{p}_{\text{total}} = mv \mathbf{i}.
\]

Because the total momentum is conserved during each of the collisions, the total momentum after each collision remains equal to \( mv \).

b) Conserve the momentum before and after each collision. Let \( v_n \) denote the speed of the composite particle after the \( n^{\text{th}} \) collision.

First collision:

\[
mv \mathbf{i} = 2mv_1 \mathbf{i} \implies v_1 = \frac{v}{2}.
\]

Second collision:

\[
2mv_1 \mathbf{i} = 3mv_2 \mathbf{i} \implies v_2 = \frac{2}{3} v_1 = \frac{2}{3} \cdot \frac{v}{2} = \frac{v}{3}.
\]

Third collision:

\[
3mv_2 \mathbf{i} = 4mv_3 \mathbf{i} \implies v_3 = \frac{3}{4} v_2 = \frac{3}{4} \cdot \frac{v}{3} = \frac{v}{4}.
\]

Fourth collision:

\[
4mv_3 \mathbf{i} = 5mv_4 \mathbf{i} \implies v_4 = \frac{4}{5} v_3 = \frac{4}{5} \cdot \frac{v}{4} = \frac{v}{5}.
\]

These speeds could have been obtained more quickly (but with less fun) by observing that after the \( n^{\text{th}} \) collision the mass of the composite and moving particle is \( (n+1)m \). Since the magnitude of the momentum is always \( mv \),

\[
(n+1)mv_n = mv \implies v_n = \frac{v}{n+1}.
\]