b) Continue the pattern of the progression. The offset of the next plank at the bottom of the stack should be \( \frac{\ell}{12} \).

9.64

a) All forces in the horizontal direction on the student-canoe system are internal forces. The total force component in the horizontal direction is zero. Hence the total momentum of the system is conserved. The momentum is initially zero and so remains zero. Hence, the center of mass of the system remains stationary.

b) Choose a coordinate system with origin where the stern (back) of the canoe was before the student began to run, and \( \hat{i} \) in the direction that the student runs. Keep the coordinate system stationary with respect to the water.

Let \( \vec{r}_{\text{canoe}} = x_{\text{canoe}} \hat{i} \) be the position vector of the center of mass of the canoe, and let \( \vec{r}_{\text{student}} = x_{\text{student}} \hat{i} \) be that of the student. Let \( m_{\text{canoe}} = 20.0 \text{ kg} \), and \( m_{\text{student}} = 80.0 \text{ kg} \) be their masses. Then the position vector for the center of mass of the student-canoe system is

\[
\vec{r}_{\text{CM}} = \frac{m_{\text{canoe}} \vec{r}_{\text{canoe}} + m_{\text{student}} \vec{r}_{\text{student}}}{m_{\text{canoe}} + m_{\text{student}}} = \frac{(20.0 \text{ kg}) x_{\text{canoe}} \hat{i} + (80.0 \text{ kg}) x_{\text{student}} \hat{i}}{20.0 \text{ kg} + 80.0 \text{ kg}} = (0.200 x_{\text{canoe}} + 0.800 x_{\text{student}}) \hat{i}.
\]

Differentiate this last expression with respect to \( t \).

(1)

\[
\frac{d}{dt} \vec{r}_{\text{CM}} = \left( 0.200 \frac{d}{dt} x_{\text{canoe}} + 0.800 \frac{d}{dt} x_{\text{student}} \right) \hat{i}.
\]

Now \( \frac{d}{dt} x_{\text{canoe}} = v_x \text{ canoe} \), the velocity component of the canoe, and \( \frac{d}{dt} x_{\text{student}} = v_x \text{ student} \), the velocity component of the student. Also, the center of mass of the system is stationary, so \( \frac{d}{dt} \vec{r}_{\text{CM}} = 0 \text{ m/s} \).

Therefore (1) becomes

\[
0 \text{ m/s} = (0.200 v_x \text{ canoe} + 0.800 v_x \text{ student}) \hat{i}.
\]

Because our reference frame is stationary with respect to the water, both of these velocity components are with respect to the water. So the last equation implies

(2)

\[
v_x \text{ canoe water} = -4.00 v_x \text{ student water}.
\]

Now use this equation in the relative velocity addition equation. In component form, we have

\[
v_x \text{ student water} = v_x \text{ student canoe} + v_x \text{ canoe water} = 3.0 \text{ m/s} - 4.00 v_x \text{ student water}
\]

\[\implies 5.00 v_x \text{ student water} = 3.0 \text{ m/s} \implies v_x \text{ student water} = 0.60 \text{ m/s}.
\]

So, \( v_x \text{ student water} = 0.60 \text{ m/s} \). Use this result in equation (2) to find

\[
v_x \text{ canoe water} = -4.00(0.60 \text{ m/s}) = -2.4 \text{ m/s}.
\]

9.65 The total momentum of the father-daughter system in the horizontal direction is conserved. Take \( \hat{i} \) to be in the direction of motion of the daughter. Let \( v_x \) be the \( x \)-component of the velocity of the father (with respect to the ice) after the parting. Then

\[
p_x \text{ total before} = p_x \text{ total after} \implies 0 \text{ kg\cdotm/s} + 0 \text{ kg\cdotm/s} = (20 \text{ kg})(2.5 \text{ m/s}) + (70 \text{ kg}) v_x \implies v_x = -0.71 \text{ m/s}.
\]

Therefore the father moves in the direction opposite to that of his daughter with a speed of 0.71 m/s with respect to the ice.

9.66