d) From part c)

$$F_{\text{lunar tidal}} = GM_{\text{Moon}}m\frac{2R}{d_{\text{Moon}}^3}$$

where M_{Moon} is the mass of the Moon and d_{Moon} is the distance between the Earth and the Moon. Similarly, the magnitude of the solar tidal force on m is

$$F_{\text{solar tidal}} = GM_{\text{Sun}}m\frac{2R}{d_{\text{Sun}}^3}$$

where M_{Sun} is the mass of the Sun and d_{Sun} is the distance between the Earth and the Sun. The ratio of the magnitudes of the two tidal forces is

$$\frac{F_{\text{lunar tidal}}}{F_{\text{solar tidal}}} = \frac{GM_{\text{Moon}}m\frac{2R}{d_{\text{Moon}}^3}}{GM_{\text{Sun}}m\frac{2R}{d_{\text{Sun}}^3}} = \frac{M_{\text{Moon}}d_{\text{Sun}}^3}{M_{\text{Sun}}d_{\text{Moon}}^3}.$$

Substitute numerical values for the masses and distances.

$$\frac{F_{\text{lunar tidal}}}{F_{\text{solar tidal}}} = \frac{(7.36\times10^{22}\text{ kg})(1.496\times10^{11}\text{ m})^3}{(1.99\times10^{30}\text{ kg})(3.84\times10^{8}\text{ m})^3} = 2.19 \approx 2.2.$$

Hence the magnitude of the lunar tidal force on m is about 2.2 times larger than the solar tidal force on m.

In fact, the Earth is the only planet in the solar system where the tidal force of the Sun is of the same order of magnitude as the tidal force from one of the planet's moons.

6.8

a) When the tidal force of the planet (mass M) on m is equal in magnitude to the gravitational force of the moon (mass M') on m, then

$$\frac{GMm\,2R}{d^3} = \frac{GM'm}{R^2}.$$

b) Solve the equation in part a) for d^3.

$$d^3 = \frac{M\,2R^3}{M'}.$$

c) Since mass is the product of density ρ and volume, the masses M' of the moon and M of the planet are

$$M' = \rho\frac{4\pi R^3}{3} \qquad \text{and} \qquad M = \rho\frac{4\pi R_0^3}{3}$$

Substitute these expressions for M and M' in the equation of part b).

$$d^3 = \frac{\rho\frac{4\pi R_0^3}{3}2R^3}{\rho\frac{4\pi R^3}{3}} = 2R_0^3 \implies d = 1.26R_0.$$

6.9

a) The magnitude of the gravitational force of the 4.00 kg sphere on the 0.400 kg sphere is

$$F = \frac{GMm}{r^2} = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(4.00\text{ kg})(0.400\text{ kg})}{(0.101\text{ m})^2} = 1.05\times10^{-8}\text{ N}.$$

b) The magnitude of the gravitational force of the 0.400 kg sphere on the 4.00 kg sphere is the same as that in part a), since they are a third law force pair.

c) Find the magnitude of the acceleration of each sphere by applying Newton's second law to each: For the 4.00 kg sphere:

$$F_{\text{total}} = ma \implies 1.05 \times 10^{-8}\ \text{N} = (4.00\ \text{kg})a \implies a = 2.63 \times 10^{-9}\ \text{m/s}^2 .$$

For the other 0.400 kg sphere:

$$F_{\text{total}} = m'a' \implies 1.05 \times 10^{-8}\ \text{N} = (0.400\ \text{kg})a' \implies a' = 2.63 \times 10^{-8}\ \text{m/s}^2 .$$

6.10

a) First find the mass of each sphere as the product of density and volume.

$$m_1 = \rho\frac{4\pi r_1^3}{3} = (11.0 \times 10^3\ \text{kg/m}^3)\frac{4\pi(5.00 \times 10^{-3}\ \text{m})^3}{3} = 5.76 \times 10^{-3}\ \text{kg},$$

and

$$m_2 = \rho\frac{4\pi r_2^3}{3} = (11.0 \times 10^3\ \text{kg/m}^3)\frac{4\pi(3.00 \times 10^{-2}\ \text{m})^3}{3} = 1.24\ \text{kg}.$$

The magnitude of the gravitational force of each sphere on the other is

$$F = \frac{GMm}{r^2} = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.76 \times 10^{-3}\ \text{kg})(1.24\ \text{kg})}{(4.00 \times 10^{-2}\ \text{m})^2} = 2.98 \times 10^{-10}\ \text{N}.$$

b) The magnitude of the weight of the smaller sphere is

$$w = m_1 g = (5.76 \times 10^{-3}\ \text{kg})(9.81\ \text{m/s}^2) = 5.65 \times 10^{-2}\ \text{N}.$$

c) The ratio is

$$\frac{w}{F} = \frac{5.65 \times 10^{-2}\ \text{N}}{2.98 \times 10^{-10}\ \text{N}} = 1.90 \times 10^8.$$

6.11 The magnitude of the gravitational force of the Earth, of mass M, on another mass m is

$$F = \frac{GMm}{r^2}.$$

where r is equal to or greater than the radius of the Earth. If this force is the only force on m, then Newton's second law applied to m is (use the magnitudes of the force and acceleration)

$$F_{\text{total}} = ma \implies \frac{GMm}{r^2} = ma.$$

Hence the magnitude of the acceleration due to gravity is

$$a = \frac{GM}{r^2}.$$

To see how the magnitude of this acceleration changes with r, take the derivative with respect to r.

$$\frac{da}{dr} = \frac{d}{dr}\left(\frac{GM}{r^2}\right) = \frac{-2GM}{r^3}.$$

Hence for a small change Δr in r, the resulting change Δa in a is given approximately by

$$\Delta a \approx \left(\frac{-2GM}{r^3}\right)\Delta r \implies \Delta r \approx -\frac{r^3}{2GM}\Delta a.$$

The change in the magnitude of the local acceleration due to gravitation is given as a 1.00% *decrease* of the value at the surface of the Earth, so

$$\Delta a = -0.0100g = -0.0981\ \text{m/s}^2 .$$

Use this in the expression for Δr above with r approximately equal to the radius of the Earth and M as the mass of the Earth.

$$\Delta r \approx \left(-\frac{(6.37 \times 10^6\ \text{m})^3}{2(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}\right)(-0.0981\ \text{m/s}^2) = 3.18 \times 10^4\ \text{m} \approx 32\ \text{km}.$$