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Currency crises, sunspots and Markov-switching regimes

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Abstract

This paper investigates the theoretical properties of a class of escape clause models of currency crises as well as their applicability to empirical work. We show that under some conditions these models give rise to an arbitrarily large number of equilibria, as well as cyclic or chaotic dynamics for the devaluation expectations. We then propose an econometric technique, based on the Markov-switching regimes framework, by which these models can be brought to the data. We illustrate this empirical approach by studying the experience of the French franc between 1987 and 1993, and find that the model performs significantly better when it allows the devaluation expectations to be influenced by sunspots. © 2000 Elsevier Science B.V. All rights reserved.

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JEL classification: F3; F4

1. Introduction

The crisis of the European Monetary System in 1992–1993, the collapse of the Mexican peso in 1994 and the Asian crises have heightened academic interest in the determinants of currency crises. Much debate has focused on whether the speculation was essentially determined by the fundamentals or whether it was, at

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least to some extent, self-fulfilling.¹ Some analytical support for the self-fulfilling view was provided by the development of new models of currency crisis that generically give rise to multiple equilibria. These models, regrouped under the name of 'escape clause' or 'second generation' approaches to currency crises, have been used extensively in recent discussions of self-fulfilling speculation (Obstfeld, 1994; Velasco, 1996; Jeanne, 1997).² One reason for the success of these models is their simplicity. But they have also been criticized for their lack of realism and robustness; Krugman (1996), in particular, presents an escape clause model that does not give rise to multiple equilibria and questions the theoretical robustness of the self-fulfilling view. Attempts at estimating escape clause models, furthermore, have been few,³ which is due in no small part to the difficulty of estimating non-linear models with multiple equilibria.

The contribution of the present paper is twofold. First, we characterize the properties of a class of escape clause models that are different from, and arguably more realistic than, those that have been considered in the literature. We show that, under certain conditions, these models can give rise to *more* equilibria than previous escape clause models, as well as cyclic or chaotic dynamics for the devaluation expectations. Second, we propose a simple empirical method by which this class of models can be brought to the data. We hope this will facilitate future empirical applications of the escape clause approach, a possibility that we illustrate by considering the experience of the French franc in 1987–1993.

The basic logic of self-fulfilling speculation in the escape clause approach is very simple. It derives from the fact that devaluation expectations increase the policymaker's desire to devalue. The most obvious way in which they do so in the real world is by raising interest rates. Faced with a dilemma between high interest rates and a devaluation, the policymaker may opt for the latter, especially if the fundamental economic situation is fragile. In fact, the policymaker may prefer a devaluation to high interest rates even though she would have maintained the fixed peg if interest rates had been low; in that case whether or not a devaluation occurs depends purely on market expectations. Escape clause models have different ways to capture this basic idea. In some models, devaluation expectations induce wage setters to predetermine high nominal wages, leading to high real wages and unemployment—unless the policymaker devalues the currency. Other models are

¹Advocates of the self-fulfilling view include Eichengreen and Wyplosz (1993) and Obstfeld and Rogoff (1995) for the EMS crisis, Cole and Kehoe (1995) for the Mexican peso crisis, and Sachs and Radelet (1998) for the Asian crisis. These authors do not maintain that speculation was purely self-fulfilling and completely unrelated to the economic fundamentals, but rather that in some of the recent episodes of crisis the effect of the economic fundamentals was augmented by self-fulfilling elements. See Obstfeld (1996b) for a clear statement of this view.

²These models have also been called 'policy optimizing' (Isard, 1995), 'endogenous policy' (Buiter et al., 1998) and 'New Crisis' models (Krugman, 1996). See Jeanne (in press) or Flood and Marion (1998) for discussions of the escape clause approach to currency crises.

³An exception is Jeanne (1997).

based on the fiscal effects of devaluation expectations. High interest rates increase the burden of public debt, inducing the policymaker to inflate and devalue rather than raising taxes. Such assumptions may seem rather special, and obviously do not capture some important channels by which devaluation expectations make themselves costly for policymakers in the real world. But, one may argue, they are meant to make the models tractable and the essence of the argument should carry over in different and more complex environments.

In a recent paper, however, Krugman (1996) argues that the insights of the escape clause literature do not survive the injection of more realism in the models' assumptions. Krugman presents a model in which devaluation expectations make themselves costly by raising the ex ante interest rate, and finds that if the fundamentals deteriorate deterministically over time multiple equilibria do not arise. The date of the crisis is uniquely determined, following a backward induction logic that is similar to the same author's 1979 article on speculative attacks — allowing him to question the theoretical specificity of the second generation approach. As noted by Kehoe (1996) and Obstfeld (1996b) in their comments on Krugman's paper, this result hinges crucially on the precise timing with which devaluation expectations affect the policymaker's decision. In Krugman's model the policymaker's decision is effectively sensitive to the devaluation expectations formed by market participants at the time of the crisis, while in previous models the same decision is dependent on the expectations formed before the crisis. This apparently innocuous difference in timing seems to alter the properties of the model to a surprising extent — a puzzle on which this paper attempts to shed some light.

The analysis of this paper is based on a framework that is a reduced form for a broad class of models, including that in Krugman (1996). We completely characterize the equilibria and give a simple criterion for their multiplicity. We show that while this class of models does not give rise to multiple equilibria when the economic fundamentals are non-stationary stochastic processes or exhibit a deterministic trend (this is a generalization of Krugman's result), they may also give rise to an *arbitrarily large* number of equilibria if a condition on the fundamentals is satisfied. This property is in sharp contrast with the models of Obstfeld (1994, 1996a), Velasco (1996) or Jeanne (1997), where the number of equilibria is no larger than three. Next, we consider a hybrid model, in which the policymaker's devaluation decision is affected by the devaluation expectations formed both in the current and previous periods. We show that in this case, the devaluations expectations are not uniquely determined in general, and that their dynamics can become cyclic or chaotic.⁴

The second part of the paper addresses the question of the empirical applicability of the escape clause approach to currency crises. We show that the class of

⁴De Grauwe et al. (1993) study chaotic dynamics in foreign exchange markets, which result in their model from mechanistic trading rules rather than being associated with rational expectations.

models that we consider in this paper can be brought to the data using a standard econometric approach, the Markov-switching regimes model developed by Hamilton and others. The Markov-switching model has been applied to a number of economic phenomena, including the business cycle (Hamilton, 1989), the term structure of interest rates (Hamilton, 1988), the dynamics of floating exchange rates (Kaminsky and Peruga, 1990; Van Norden, 1996), and more recently currency crises (Martinez-Peria, 1998; Piard, 1997; Psaradakis et al., 1998). We show here that a linearization of our model gives a Markov-switching regimes model for the devaluation probability, in which the switch across regimes corresponds to jumps between different equilibria. This provides some theoretical justification for the use of the Markov-switching regimes approach in empirical work on currency crises, and can also help to assess the empirical plausibility of the multiple equilibria hypothesis. To illustrate, we estimate a Markov-switching regimes model for the French franc over the period 1987-1993, and find that a model allowing for sunspots performs better than a purely fundamental-based model, in particular by improving the relationship between the economic fundamentals and the devaluation expectations.

The paper is structured as follows. Section 2 presents the model and investigates its properties. Section 3 relates our model with the Markov-switching regimes model, which is estimated on French data. Section 4 concludes.

2. The model

This section presents a stylised model of a fixed exchange rate peg. Like in Krugman (1996) or Morris and Shin (1998), the model is essentially a reduced form representation of the policymaker's decision whether or not to defend the fixed peg. After a statement of the assumptions in Section 2.1, we study the equilibria in which devaluation expectations are determined uniquely by the fundamentals in Section 2.2, before examining the conditions under which self-fulfilling speculation might arise (Section 2.3). Section 2.4 scrutinizes the possibility of cyclic and chaotic dynamics in the devaluation expectations.

2.1. Assumptions

Consider a country that has committed to a fixed exchange rate peg, but can at each period exercise an escape clause and devalue. The domestic policymaker decides whether or not to devalue by comparing the benefits and costs of maintaining the fixed peg. She devalues if the net benefit of the fixed peg is negative. We assume that the net benefit of the fixed peg at time *t* can be written in reduced form:

$$B(\phi_{t}, \pi_{t}) \tag{1}$$

where ϕ_t is a variable reflecting the exogenous economic fundamentals,⁵ and $\pi_t = \int_0^1 \pi_t^i di$ is the average estimate at t of the probability of a devaluation at t+1 formed by a continuum of atomistic speculators $i \in [0,1]$. We assume that the net benefit of the fixed peg is a continuously differentiable function of both variables, increasing with the level of the fundamental and decreasing with the devaluation probability $(B_1 > 0, B_2 < 0)$. We also make the (technical) assumption that whatever the level of the devaluation probability, there is a level of the fundamental at which the policymaker is indifferent between devaluing or not, i.e. $\forall \pi, \exists \phi, B(\phi, \pi) = 0$.

This formulation is meant to represent in a compact way the idea that, while the net benefit of a fixed peg depends on the economic fundamentals, it is also sensitive to devaluation expectations through the level of interest rates. Other things equal, higher devaluation expectations mean that the monetary authorities must set the interest rate at a higher level, which makes the fixed peg more costly through a number of channels (lower economic activity, fragilization of the banking sector, higher interest burden on the public debt, etc.). Krugman (1996) presents a simple model in which devaluation expectations depress output by raising the ex ante interest rate, and the net benefit of the fixed peg for the policymaker can be written like Eq. (1) in reduced form.

The dynamics of the system are driven by the exogenous fundamental variable, ϕ . We assume that this variable is stochastic, and that its movements are well described by a Markov process with a transition cumulative distribution function $F(\cdot, \cdot)$:

$$F(\phi, \phi') = \text{Prob}[\phi_{t+1} < \phi' | \phi_t = \phi] \tag{2}$$

We assume $F_1 \le 0$, which may be interpreted as a requirement that the fundamental not be negatively autocorrelated (in the sense that an increase in the current value of the fundamental shifts the cumulative distribution function of the next period fundamental in the same direction).

The devaluation probability is the endogenous variable of the model. In order to understand how it is determined, let us first consider the problem at the level of an individual atomistic speculator, i, who makes his own assessment of the devaluation probability, π^i , taking as given the expectations of other speculators. Being rational, the speculator will estimate the devaluation probability as the mathematical probability that the net benefit of the fixed peg will be negative in the next period:

$$\pi_t^i = \text{Prob}[B(\phi_{t+1}, \pi_{t+1}) < 0 | \phi_t]$$
 (3)

⁵Variable ϕ_t reflects all the exogenous economic factors influencing the policymaker's decision whether or not to devalue at date t, including the past values or the expected future values of the economic fundamentals.

where the probability is assessed conditionally on the current level of the fundamental variable. This equation shows a property which is quite important for the logic of self-fulfilling speculation in this model: the expectations of a rational speculator are forward looking, and depend not only on the speculator's beliefs about the future fundamentals but also on his beliefs about the future beliefs of other speculators. A rational speculator knows that the expectations of other speculators will influence the cost of maintaining the fixed peg at the next period and so the objective probability of a devaluation.

Assuming that all the speculators are rational and share common knowledge of the same information set, 6 we can drop index i and write the devaluation probability estimated by the representative speculator at time t as:

$$\pi_{t} = \text{Prob}[B(\phi_{t+1}, \pi_{t+1}) < 0 | \phi_{t}]$$
 (4)

This equation summarizes the relationship between the fundamentals and the devaluation expectations implied by the model assumptions. Characterizing the equilibrium devaluation expectations means finding the stochastic processes π that are solutions to Eq. (4) for a given exogenous process of the fundamental, ϕ .

2.2. Fundamental-based equilibria

In a fundamental-based equilibrium the state of the economy is uniquely determined by the exogenous fundamental ϕ_i . There is a critical level of the fundamental, ϕ^* , under which the policymaker opts out, and above which she maintains the fixed peg. This level is determined as a fixed point in the mappings between the speculators' expectations and the policymaker's policy. Let us denote by ϕ^{*e} the level of the fundamental under which speculators expect the policymaker to devalue. Then each speculator estimates the devaluation probability at time t as the probability that the fundamental will fall short of ϕ^{*e} at the following period:

$$\pi_t = \operatorname{Prob}[\phi_{t+1} < \phi^{*e} | \phi_t] = F(\phi_t, \phi^{*e}) \tag{5}$$

Conversely, the policymaker's problem is to determine the optimal triggering level of the fundamental given the speculators' expectations. The level chosen by the policymaker, ϕ^* , is such that the net benefit function:

⁶The assumption of common knowledge is not innocuous. As Morris and Shin (1998) have shown in a recent paper, the absence of common knowledge can remove the multiplicity of equilibria in escape clause models of currency crises.

$$\phi \mapsto B(\phi, F(\phi, \phi^{*e})) \tag{6}$$

takes negative values for ϕ lower than ϕ^* and positive values for ϕ larger than ϕ^* . Since the net benefit function is a strictly increasing function of the fundamental, ϕ^* is the (unique) level of the fundamental at which the net benefit is equal to zero. We denote by $H(\phi^{*e})$ this level.

In a rational expectations equilibrium the beliefs of speculators must be true, i.e. ϕ^* must be a fixed point of function $H(\cdot)$:

$$\phi^* = H(\phi^*) \tag{7}$$

This equation says that the level of the fundamental under which speculators expect the policymaker to devalue is the same as the level under which the policymaker effectively chooses to devalue. It always has one solution, which ensures the existence of at least one fundamental-based equilibrium. But it may also have multiple solutions. To illustrate, Fig. 1 shows a case where there are three possible levels of the critical benefit threshold $\phi_1^* < \phi_{11}^* < \phi_{111}^*$. This multiplicity is made possible by the fact that function $H(\cdot)$ is increasing, or in other words, that there is a *strategic complementarity* between the market expectations about the policymaker's devaluation rule and the rule that is actually chosen by the policymaker. By increasing their estimate of the critical threshold triggering the devaluation ϕ^{*e} , speculators force the policymaker to bear the cost of higher devaluation expectations, inducing her to revise the actual threshold ϕ^* upwards. As a result, fundamental-based equilibria with different devaluation rules—and different average levels of devaluation expectations—may coexist.

2.3. Sunspot equilibria

The multiplicity of fundamental-based equilibria makes it possible to construct equilibria in which the economy jumps across states with different levels of devaluation expectations. A priori, the jumps between states may be related to the

⁷The *existence* of $H(\phi^{*e})$ is ensured by the following argument. The net benefit function Eq. (6) is bounded from below by $B(\phi, 1)$ and from above by $B(\phi, 0)$. Because of our technical assumption on $B(\cdot, \cdot)$ we know that there exist ϕ^0 and ϕ^1 such that $B(\phi^0, 0) = B(\phi^1, 1) = 0$. It is then not difficult to check that, by continuity of the net benefit function, there is at least one ϕ^* , between ϕ^0 and ϕ^1 , for which the net benefit is zero.

⁸This results from the fact that $H(\cdot)$ is continuous and bounded by ϕ^0 and ϕ^1 .

⁹Fig. 1 was constructed assuming that the net benefit function is given by $B(\phi, \pi) = 1 + 0.3\phi - 2\pi$ and that ϕ_i is identically, independently and normally distributed, centered on $\phi = 1$ with variance 0.25. For well-behaved fundamental processes—that is, in which the innovation has a probability distribution function that is bell-shaped and symmetric—the number of solutions is limited to three, but it can be larger in general.

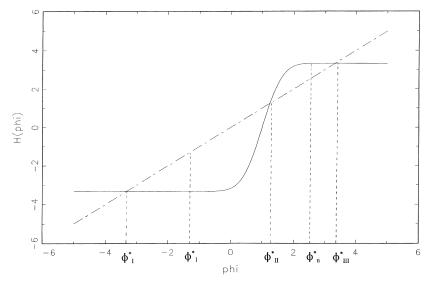


Fig. 1. Solution for ϕ^* .

fundamentals, but this is not necessarily the case; they may also be driven by extrinsic uncertainty—a sunspot variable which coordinates the private sector expectations on one state or the other. We now proceed to construct such sunspot equilibria.

A sunspot equilibrium is formally defined as follows. We assume that the economy can be in n states $s=1,\ldots,n$, which differ from each other by the level of the fundamental triggering devaluation. We assume that if the state at time t is s, the policymaker opts out if and only if $\phi_t < \phi_s^*$. The threshold fundamental levels are ranked by increasing order, i.e. $\phi_1^* < \phi_2^* < \cdots < \phi_n^*$, which means that if the policymaker devalues when the state is s, she also devalues for any state higher than s. Like in Jeanne (1997), the transition across states is assumed to follow a Markov process independent of the fundamentals, characterized by the transition matrix $\Theta = [\theta(i, j)]_{1 \le i, i \le n}$.

Two clarifying remarks are worth making at this juncture. First, it is important to note that the jumps in ϕ^* do not reflect any change in the policymaker's preferences or type. They correspond to changes in the policymaker's *decision rule* that are induced purely by shifts in the speculators' expectations. Second, the ϕ^*_s are a priori not the same as the critical thresholds of the fundamental-based equilibria, i.e. the ϕ^* solutions to Eq. (7). This equation does not take into account the speculators' expectations about future state shifts, which play an important role in shaping sunspot equilibria.

In a sunspot equilibrium the devaluation probability depends jointly on the state and the fundamental variable. It is equal to the sum of the probabilities of a devaluation in the next period weighted by the transition probabilities from the current to the future states, i.e.:

$$\pi_t = \sum_{s=1}^n \theta(s_t, s) F(\phi_t, \phi_s^*) \tag{8}$$

Given these expectations, the net benefit function of the policymaker now depends jointly on the current state, the probabilities of a transition to other states and the corresponding fundamental threshold levels. In state s the net benefit function is given by:

$$\phi \mapsto B\left(\phi, \sum_{s'=1}^n \theta(s, s') F(\phi, \phi_{s'}^*)\right)$$

Again, the policymaker chooses ϕ_s^* as the unique level of ϕ for which the net benefit is equal to zero. We denote by $H_s(\phi_1^*,\ldots,\phi_n^*)$ this level, which, in a rational expectations equilibrium, should satisfy the fixed point equations:

$$\forall s = 1, \dots, n, \quad \phi_s^* = H_s(\phi_1^*, \dots, \phi_n^*)$$
 (9)

We characterize a sunspot equilibrium by a vector $(\phi_1^*, \ldots, \phi_n^*)'$ that satisfies the n constraints Eq. (9). One can note that the fundamental-based equilibria may be viewed as degenerate cases of the sunspot ones, corresponding to Θ equal to the identity matrix. In that case, the economy never jumps, and always remains in its initial state. Eq. (9) reduces to Eq. (7) for all states s, and the ϕ_s^* are necessarily equal to ϕ_1^* , ϕ_{11}^* or ϕ_{111}^* . Of course, we are more interested in non-degenerate cases, in which the economy actually jumps between different states, and it is the latter type of sunspot equilibria on which we focus henceforth.

Proposition 1 gives a simple criterion for the existence of sunspot equilibria (see proof in Appendix A).

Proposition 1. Sunspot equilibria exist if and only if there are multiple fundamental-based equilibria, i.e. multiple solutions to Eq. (7). Moreover, if this condition is satisfied, it is possible to construct sunspot equilibria with any number of states n.

One might have expected the number of states to be the same as the number of solutions to Eq. (7). The last part of Proposition 1 shows this conjecture to be wrong. In fact, the number of states can be arbitrarily large. This implies that we can take the states arbitrarily close to each other, and in the limit define the set of states as a continuum. The intuition (and the proof of Proposition 1) relies on the fact that in a given sunspot equilibrium it is always possible to 'stack' new states between the existing ones. We show in the proof how one can construct a new state as a convex combination of two existing states.

This property is in sharp contrast with the second-generation models of Obstfeld (1994, 1996), Jeanne (1997) and Velasco (1996), where the number of states is no larger than three. The difference comes from the assumptions concerning the timing of devaluation expectations. In other papers, the net benefit of the fixed exchange rate system at a given period depends on the devaluation expectations formed in the preceding period. In our reduced-form notation, this corresponds to the assumption that the net benefit at time t can be written $B(\phi_t, \pi_{t-1})$, so that Eq. (4) is replaced by

$$\pi_t = \operatorname{Prob}[B(\phi_{t+1}, \pi_t) < 0 | \phi_t] \tag{10}$$

This equation can have multiple solutions since both sides are increasing with π_t . But it is a closed-loop equation that involves the value of the devaluation probability at period t only, and for well-behaved fundamental processes the number of possible values for the devaluation probability is no larger than three. By contrast, our model, like Krugman's (1996) one, assumes that the net benefit of the fixed peg depends on the current period expectations about the future, which makes the determination of the devaluation probability an open-loop problem and enlarges considerably the set of equilibria.

Whether or not sunspot equilibria exist depends on the shape of function $H(\cdot)$, which in turn depends in a complex way on the policymaker's net benefit function and the stochastic process followed by the fundamental. It is possible, however, to state a condition on the fundamental process that is necessary for the multiplicity of equilibria. This condition is related to $F(\phi, \phi)$, the probability that the fundamental will be lower than ϕ the next period when it is equal to ϕ in the current period, or in other words, the probability of a decrease in the fundamental.

Corollary 1. For sunspot equilibria to exist the probability of a decrease in the fundamental, $F(\phi, \phi)$, must be strictly increasing with the fundamental, ϕ , at least over some range.

To see why the corollary is true, let us consider two different fundamental-based equilibria, say I and II. In equilibrium II the point where the policymaker is indifferent between devaluing and maintaining the fixed peg is reached for a higher level of the fundamental than in equilibrium I. But in both equilibria, when the fundamental is exactly equal to the threshold level triggering a devaluation, the devaluation probability is exactly the same as the probability of a decrease in the fundamental between the current and next period. Hence it must be the case that:

$$F(\phi_{I}^{*}, \phi_{I}^{*}) \leq F(\phi_{II}^{*}, \phi_{II}^{*})$$

which is possible only if $F(\phi, \phi)$ is strictly increasing with ϕ , at least over some range.

As a negative corollary to the previous result, one can derive a number of conditions under which self-fulfilling speculation *cannot* arise in our model.

Corollary 2. Assume that one of the following assumptions is satisfied: (i) the fundamental variable is always decreasing, i.e. $\operatorname{Prob}_t[\phi_{t+1} < \phi_t] = 1$; (ii) the fundamental variable is always increasing i.e. $\operatorname{Prob}_t[\phi_{t+1} < \phi_t] = 0$; (iii) the fundamental variable follows a random walk, with $\operatorname{Prob}_t[\phi_{t+1} < \phi_t] = 1/2$; then sunspot equilibria do not exist.

The proof of the corollary is that $F(\cdot, \cdot) = 1, 0, 1/2$ in cases (i), (ii) and (iii), respectively, so that the condition stated in Corollary 1 is not satisfied. An implication of this corollary is Krugman's finding that his model does not give rise to multiple equilibria when the fundamental follows a downward deterministic trend. Krugman obtains this result by showing that if the policymaker is sure to devalue before a finite date, the effective devaluation date is uniquely determined by backward induction. The corollary shows that this result can be generalized to the case when the fundamental is always deteriorating but is stochastic, when it always improves over time, or follows a random walk.

2.4. A digression on cycles and chaos

As the previous section shows, the properties of an escape clause model of currency crisis are very sensitive to whether the net benefit of the fixed peg at time t is affected by the devaluation expectations formed at time t or at time t-1. The first assumption would be natural if devaluation expectations were costly because of their impact on nominal wage-setting (as in Obstfeld, 1994, 1996a), or the expost or lagged ex ante real interest rate (as in Eichengreen and Jeanne, 1998). The second assumption relies on the view that devaluation expectations matter because of their impact on the current ex ante real interest rate (Krugman, 1996). In the real world these channels are probably important simultaneously, which raises the question of the properties of a second-generation model in which the net benefit of the fixed peg at time t depends on both π_t and π_{t-1} . We show in this section that the dynamics of devaluation expectations become more complicated, and may exhibit cyclic or chaotic features.

We study a simple example based on a linear specification for the benefit function $B(\cdot, \cdot)$ and a fundamental variable that is independently and identically distributed over time

$$B(\phi_t, \pi_t) = b_0 + b_1 \phi_t - b_2 \pi_t - b_3 \pi_{t-1}$$
(11)

$$\phi_t = \overline{\phi} + \epsilon_t \tag{12}$$

Then:

$$\pi_t = \operatorname{Prob}[B_{t+1} < 0] \tag{13}$$

$$=G\left(\frac{-b_0 - b_1\overline{\phi} + b_2\pi_{t+1} + b_3\pi_t}{b_1}\right) \tag{14}$$

where $G(\cdot)$ is the cumulative distribution function of ϵ . Hence, the dynamics of the devaluation probability are deterministic of the first order, and characterized by:

$$\pi_{t+1} = \frac{b_1 G^{-1}(\pi_t) + b_0 + b_1 \overline{\phi} - b_3 \pi_t}{b_2}$$
 (15)

Fig. 2 depicts a possible shape for this relationship, obtained under the assumption that ϵ is normally distributed. The intersection of the curve and the 45° line defines a level of the devaluation probability which is a fixed point of the expectational problem. However, this equilibrium is unstable because the slope of the curve at its intersection with the line is less than -1. Starting from a level of the devaluation probability to the left or the right of the fixed point gives rise to chaotic dynamics, illustrated in Fig. 3. It is also possible to find parameter values which, by making the slope of the curve at its intersection with the 45° line closer to -1, make the dynamics of the devaluation probability cyclic.

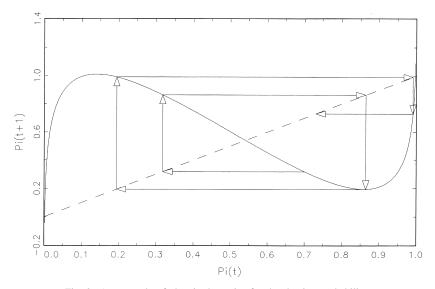


Fig. 2. An example of chaotic dynamics for devaluation probability.

¹⁰Figs. 2 and 3 were obtained for the specification $\pi_{r+1} = 0.76G^{-1}(\pi_r) + 2.31 - 3.415 \pi_r$, where $G(\cdot)$ is the c.d.f. of a standard normal with unit variance.

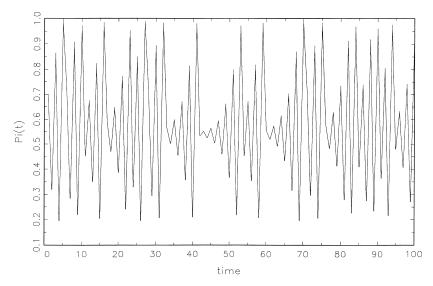


Fig. 3. Time series for $\Pi(t)$.

3. Multiple equilibria and Markov-switching regimes

We now proceed to the question of the empirical implementability of the escape clause approach to currency crises. The empirical literature provides ample evidence that devaluation expectations are subject to abrupt shifts that do not seem related to the economic fundamentals. This evidence has been presented by some authors in the context of the Markov-switching regimes model developed by Hamilton and others. The regime shifts are then interpreted as jumps between multiple equilibria, even though, of course, Hamilton's framework is not a structural model of multiple equilibria. We show in this section that a Markov-switching regimes model of the devaluation expectations can in fact be interpreted as a linearized reduced form of our structural model with sunspots (Section 3.1). We then illustrate the potential application of this equivalence result to empirical work by considering the experience of the French franc (Section 3.2).

3.1. A structural interpretation of Markov-switching regimes models

Let us consider a sunspot equilibrium of the model presented in Section 2, with n states and n threshold levels $\phi_1^* < \cdots < \phi_n^*$. We assume that the fundamental variable is a linear index aggregating the macroeconomic variables that are most relevant for the policymaker's choice of maintaining or not the fixed peg, plus a shock:

$$\phi_t = \alpha' x_t + \eta_t$$

where $\alpha = (\alpha_1, \dots, \alpha_K)'$ is a vector of coefficients, $x_t = (x_{1t}, \dots, x_{Kt})'$ is a vector of relevant economic fundamentals, and η is an i.i.d. stochastic term reflecting other exogenous determinants of the policymaker's behavior.

We then linearize the model under the assumption that the fluctuations of the fundamental variable and the differences between the critical thresholds are small, i.e.:

$$\phi_t = \overline{\phi} + \delta \phi_t$$

$$\phi_s^* = \phi^* + \delta\phi_s^*$$

where $\delta \phi_t$ and $\delta \phi_s^*$ are of the first order.

Linearizing the equation for the devaluation probability, Eq. (8), gives:

$$\pi_t = \gamma_{s_t} + \beta' x_t + \nu_t, \quad s_t = 1, \dots, n$$
 (16)

where γ_s is a constant that depends on the state, $\beta = (\beta_1, \dots, \beta_K)'$ is a vector of coefficients and ν_t is an i.i.d. shock, all of which can be written as functions of the structural parameters of the model.¹¹

Eq. (16) may be viewed as a Markov-switching model with n regimes. Regime shifts affect the devaluation probability by changing the constant term on the right-hand side of the equation, but leave the coefficients of the fundamentals unchanged—a restriction that is not usually adopted in Markov-switching regimes models. These regime shifts can be interpreted as jumps between different states of market expectations in the underlying model with sunspots. A jump to a state of higher devaluation expectations makes the devaluation more likely and increases the constant term γ .

The likelihood of the Hamilton model is defined in the same way as the likelihood of the structural model with sunspots. In the degenerate case where there is only one state, Hamilton's model reduces to a simple linear regression of the devaluation probability on economic fundamentals, of the type estimated, e.g. by Rose and Svensson (1994). Several papers have explored how Markovswitching models with several regimes can be estimated using the maximum likelihood method, and the methods that they develop can easily be transposed to our setting.¹²

The formula are: $\gamma_s = F(\overline{\phi}, \phi^*) + F_2(\overline{\phi}, \phi^*) \sum_{s'=1}^n \theta(s, s') \delta \phi_{s'}^* - F_1(\overline{\phi}, \phi^*) \overline{\phi}, \beta = F_1(\overline{\phi}, \phi^*) \alpha$ and $\nu_i = F_1(\overline{\phi}, \phi^*) \eta_i$.

¹²It should be noted, however, that the maximum likelihood estimation of the Hamilton model is not the same as the maximum likelihood estimation of the underlying escape clause model because the former does not take into account all the structural constraints that arise in the latter. We come back to that point at the end of Section 3.2.

3.2. Empirical illustration: the French franc, 1987–1993

We illustrate the equivalence between our model and a Markov-switching regimes model by considering the example of the French franc. Some authors have argued that the speculation against the franc was self-fulfilling in 1992–1993 (see, e.g. Eichengreen and Wyplosz, 1993), and the experience of the franc has later been used as a benchmark case study of self-fulfilling speculation (Jeanne, 1997; Martinez-Peria, 1998; Piard, 1997; Psaradakis et al., 1998). Moreover the franc offers the advantage of providing a long sample period with many speculative episodes but without change of regime.

We estimated the model of Eq. (16) with two states. Our dependent variable is an estimate of the devaluation probability, in %, measured as the one-month interest differential between Euro-franc and Euro-DM instruments, after correcting for expected movement toward the center of the band using the drift adjustment method of Svensson (1993), and assuming a devaluation size of 5% (roughly the size of the average realignment of the franc in the 1979–1986 period). Our sample includes monthly data between February 1987 and July 1993, which is the longest sample period without change in regime for the franc (it starts after the last franc devaluation, which took place in January 1987, and ends before the ERM band was widened to 15% in August 1993).

A key choice is the set of fundamentals. Traditional measures of exchange rate overvaluation or undervaluation focus on the balance of payments and relative prices or costs. The ERM crisis also led to consideration of a wider set of fundamentals. In second generation currency crisis models, other variables (growth, unemployment, the health of the banking sector) which may appear in the authorities' objective function are obviously relevant in forming devaluation expectations. We therefore include among the fundamentals the unemployment rate (*ur*), as well as the trade balance (as ratio to GDP, *trbal*) and the percentage deviation of the real effective exchange rate from its 1990 level (*rer*). The real exchange rate is computed on the basis of unit labor costs in production; an

¹³One problem with using the drift-adjustment method is that it is derived from an implicit target zone model which is not the same as—though not necessarily inconsistent with—the escape clause model that we focus on. Introducing some features of target zones models into the escape clause approach to currency crises is an interesting research topic of its own, which goes beyond the scope of the present paper.

¹⁴Data are taken from the International Financial Statistics (IMF). The set of fundamentals could of course be widened further, in particular to include fiscal variables, which are critical in many speculative attack models because they explain domestic credit and hence monetary growth. However, as in most other developed economies, there is no automatic mechanism in France linking deficits to money creation, and seigniorage over this period was negligible. Moreover, the deterioration of the deficit over our sample period was largely due to cyclical factors (which are also reflected in the unemployment rate), and the public debt ratio, which remained below 50% of GDP, was not likely to have been a factor in explaining interest rates in France (unlike in Italy, where it rose to 120% of GDP).

increase in this index corresponds to a real appreciation of the franc. A time trend (t) is also included as a short cut to capture reputational considerations. Maintaining a fixed parity in the EMS has been justified by the desire of the policymaker to acquire an anti-inflationary reputation (e.g. Giavazzi and Giovannini, 1989), and such reputation plausibly builds gradually through time as private agents revise their beliefs about the policymaker using Bayesian learning (Masson, 1995). Variables tbal and rer are plotted in Fig. 4, while ur and the growth rate of output are plotted in Fig. 5.

The equation for the devaluation probability was specified as:

$$\pi_{t} = \gamma_{s_{t}} + \beta_{u}ur_{t} + \beta_{t}t + \beta_{b}trbal_{t} + \beta_{r}rer_{t} + \nu_{t}$$

where the value of the constant term depends on the state, with $s_t = 1$ or 2, and the error term ν was taken to be normally distributed, with variance σ_{ν}^2 . Transition between states was assumed to be governed by a Markov process, characterized by a 2×2 matrix of transition probabilities Θ . The initial state also needs to be estimated, introducing a parameter $\mu = \Pr_0(s_1 = 1)$.

Estimation proceeded by first estimating the model without multiple equilibria (i.e. the purely fundamentals-based model), which can be done with ordinary least

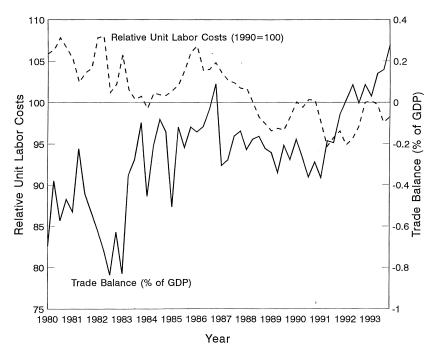


Fig. 4. Relative unit labor costs and trade balance, 1980-1993.

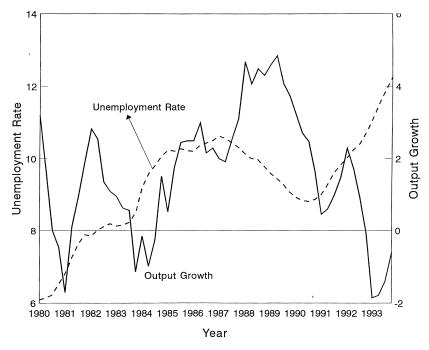


Fig. 5. Unemployment rate and output growth, 1980-1993 (%).

squares. The results are presented in column (1) of Table 1, and the forecast values for the probability of devaluing are plotted against the data in Fig. 6. The results have the expected signs for all variables except for the trade balance (a larger surplus for France should narrow the interest differential, not widen it). From Fig. 6 it can be seen that though the fitted values track the broad trend of π_i , they do not capture any of the movements associated with episodes of speculation.

Then the model was estimated with two states. Following Hamilton (1994), the EM algorithm was programmed in Gauss to get close to maximum likelihood estimates, and then Gauss's MAXLIK procedure was used to get the final estimates.¹⁵ The estimates of the two-state model, presented in column (2) of Table 1, are more satisfactory in several respects. First, the fit of the model is considerably better, as evidenced by a lower σ_{ν} (less than half the previous one), a higher log likelihood, and substantially different values of γ in the two states. The difference in log likelihoods (multiplied by two times the number of observations, 78), yields a test statistic of 106.58. A formal test is complicated by the fact that several parameters (in particular, the probabilities of being in the different states)

¹⁵As noted in Van Norden and Vigfusson (1996), the EM algorithm comes close to yielding maximum likelihood estimates, but does not quite reach the maximum.

lnL/T

-1.0388

			`	1	
	(1)	(2)		•	
$\gamma_{_{1}}$	4.969	0.199			
	(0.194)	(2.114)			
γ_2	_	3.818			
		(2.088)			
$\beta_{\!\scriptscriptstyle u}$	0.183	0.544			
	(0.371)	(0.186)			
$oldsymbol{eta_{\scriptscriptstyle t}}$	-0.0790	-0.0474			
	(0.0135)	(0.0086)			
$oldsymbol{eta}_{\!\scriptscriptstyle b}$	1.055	-0.425			
	(0.437)	(0.254)			
β_r	0.163	0.0635			
	(0.171)	(0.0920)			
$\sigma_{_{\!\scriptscriptstyle u}}$	1.714	0.834			
μ	_	1.00			

Table 1
Maximum likelihood estimates of parameters (standard errors in parentheses)

-0.3556

are not defined under the null of a single regime (Hamilton, 1994; Hansen, 1996). Thus it is not legitimate simply to compare the difference in log likelihoods to a χ^2 with 1 degree of freedom, whose critical value at the 1% level is 6.63. An overly conservative approach would be to allow fully for the 3 extra degrees of freedom, and compare the difference in log likelihoods to a $\chi^2(4)$, whose 1% critical value is 13.28. Even this is vastly exceeded. Second, each coefficient now has its

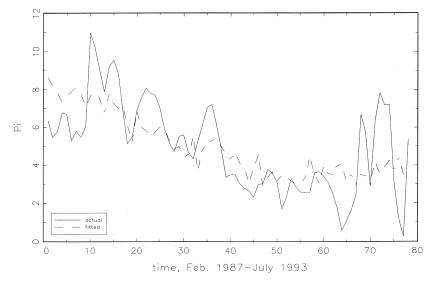


Fig. 6. One-state model: Probability of devaluation, actual and fitted.

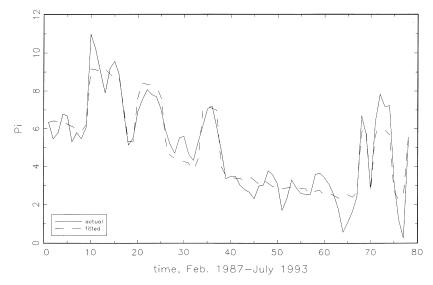


Fig. 7. Two-state model: Probability of devaluation, actual and fitted.

expected sign, including the trade balance, and all except the real exchange rate are asymptotically significant at the 10% level. Third, the plot in Fig. 7 shows that the model with multiple equilibria seems to capture well several of the episodes of sharp movements in the devaluation probability. In particular, the sharp upward moves around t = 10, t = 20 and t = 35, as well as the upticks after t = 67 (August 1992), are modelled by a jump to the second equilibrium.

The estimated Θ matrix of transition probabilities

$$\Theta = \begin{pmatrix} 0.771 & 0.229 \\ 0.145 & 0.855 \end{pmatrix}$$

shows that both states are fairly stable. This is illustrated in Fig. 8, which gives the smoothed probability estimates of being in the first state. There is clearly some persistence in the behavior of the devaluation probability, which tends to stay in one state or the other for several periods.

It should be noted that there is a sense in which the estimation of the Hamilton filter is less restricted than the estimation of the underlying escape clause model with multiple equilibria. We have *assumed*, in order to derive the former, that the underlying escape clause model had multiple equilibria. It remains to be seen whether the fundamental process that results from the estimation warrants such an assumption. Developing the techniques that would allow us to do so is a topic for future research. We now conclude the paper by outlining other interesting directions of research, after a brief summary of our results.

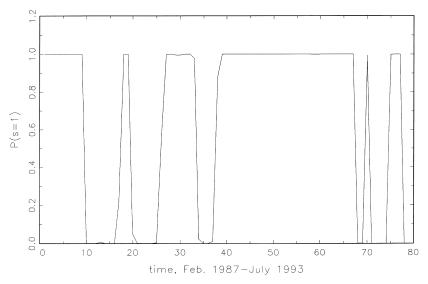


Fig. 8. Two-state model: Smoothed probability of being in state 1.

4. Concluding comments

This paper has investigated the properties of a class of escape clause models of currency crises in which the policymaker's objective function is influenced by the current period devaluation expectations of speculators. We found that, contrary to Krugman's (1996) claim, these models are not inconsistent with multiple equilibria, and can even give rise to a richer set of equilibria than other escape clause models of currency crises. We also showed that the model is amenable to empirical analysis using a standard econometric technique in time series analysis, the Markov-switching regimes model. We found that the model gives a substantially better account of the recent experience of the French franc when it gives a role to sunspots, in particular by tracking better the episodes of speculation—interpreting them as self-fulfilling jumps in the beliefs of foreign exchange market participants. It would be interesting to study whether the better performance of the sunspot model is an empirical regularity that holds for currencies other than the franc.

However, we would hasten to acknowledge that there remains considerable scope for further development. Even those economists who support the thesis of self-fulfilling speculation express some dissatisfaction with the state of the art of modeling multiple equilibria. In particular, the assumption that the economy jumps from one equilibrium to another following the realization of an extraneous shock raises a number of questions. To the extent that the sunspot variable instantaneously coordinates the expectations of all market participants, one would like to relate

this variable to an event that is publicly observable. It would be interesting, in this respect, to see whether the transitions between states that are identified by the Markov-switching technique are correlated with political events or other news, but this would require extending the analysis to a higher data frequency than monthly. A more radical criticism is that the selection of the equilibria should not be based on an hypothetical variable, but rather on an explicit modeling of the dynamics of the beliefs of heterogeneous market participants. From this point of view, it would be interesting to see what the approach of Morris and Shin (1998) can teach us about the determination of equilibria in our setting.

Finally, some extensions of our model have potentially interesting properties, like cyclical or chaotic dynamics, that we have only touched upon in this paper. It is noteworthy that in our model, these chaotic dynamics are perfectly consistent with the rationality of the foreign exchange market participants, and in particular do not require some of them to follow ad hoc trading rules. Whether these non-linear dynamics give a good account of devaluation expectations is an interesting question for future research.

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Appendix A. Proof of Proposition 1

We first consider a sunspot equilibrium characterized by a vector $(\phi_n^*, \ldots, \phi_n^*)'$ with $\phi_1^* < \cdots < \phi_n^*$ and a Markov matrix Θ , and show that Eq. (7) must have multiple solutions. Assume that the economy is in state 1. If speculators were sure that the state remained 1 in the next period, the policymaker's devaluation threshold would be $H(\phi_1^*)$. But in a sunspot equilibrium the probability that the economy shifts to higher states in the next period raises speculators' devaluation expectations, and increases the fundamental threshold chosen by the policymaker to a level, $H_1(\phi_1^*, \ldots, \phi_n^*)$, which is higher than $H(\phi_1^*)$. Hence:

$$\phi_1^* = H_1(\phi_1^*, \ldots, \phi_n^*) > H(\phi_1^*)$$

and similarly one can show that $\phi_n^* = H_n(\phi_1^*, \dots, \phi_n^*) < H(\phi_n^*)$. Then Fig. 1

makes clear that $H(\phi_1^*) < \phi_1^*$ and $H(\phi_n^*) > \phi_n^*$ can be consistent with $\phi_1^* < \phi_n^*$ if and only if there are multiple solutions to Eq. (7), and $\phi_1^* \in]\phi_1^*$, $\phi_{II}^*[$ and $\phi_n^* \in]\phi_1^*$, $\phi_{II}^*[$.

We now show that it is always possible to add a new state to a given sunspot equilibrium. This will prove that the multiplicity of solutions to Eq. (7) is not only necessary but also sufficient, by showing how it is possible to construct a sunspot equilibrium by adding states between the fundamental-based equilibria. It will also prove, by induction, that the number of states can be arbitrarily large.

We construct an additional state as a convex combination of an arbitrarily chosen pair of states. For the sake of notational convenience, we consider states 1 and 2, and denote by 3/2 the new intermediate state. We choose arbitrarily a fundamental threshold $\phi_{3/2}^*$ between ϕ_1^* and ϕ_2^* and construct a new state with this threshold by choosing appropriate transition probabilities.

We need to find a $(n+1) \times (n+1)$ Markov matrix Θ' that satisfies Eq. (9) for states $s=1, 3/2, 2, \ldots, n$. Let us assume that in the new equilibrium the transition probabilities involving states other than 1, 3/2, and 2 are unchanged, i.e. $\forall s$ and $s' \neq 3/2$, $\theta'(s, s') = \theta(s, s')$ if s or $s' \notin \{1, 2\}$. We also assume that the economy can jump to state 3/2 only from state 1 or 2, i.e. $\forall s \notin \{1, 3/2, 2\}, \theta'(s, 3/2) = 0$. Then Eq. (9) is satisfied for all s different from 1, 2 and 3/2, so that we can restrict the attention to the latter states. One must find transition probabilities such that the net benefit is equal to 0 when $\phi = \phi_s^*$ in each state s=1, 3/2, 2. Let us first consider states 1 and 2. The sum of the transition probabilities from states 1 and 2 must remain unchanged:

$$\theta'(s,1) + \theta'(s,3/2) + \theta'(s,2) = \theta(s,1) + \theta(s,2) \tag{A.1}$$

and the introduction of the new state should not change the devaluation probability when the fundamental is equal to the threshold level, so that:

$$\theta'(s, 1)F(\phi_s^*, \phi_1^*) + \theta'(s, 3/2)F(\phi_s^*, \phi_{3/2}^*) + \theta'(s, 2)F(\phi_s^*, \phi_2^*)$$

$$= \theta(s, 1)F(\phi_s^*, \phi_1^*) + \theta(s, 2)F(\phi_s^*, \phi_2^*)$$

for each state s = 1, 2. It is not difficult to find $\theta'(s, 1), \theta'(s, 3/2)$ and $\theta'(s, 1)$ between 0 and 1 satisfying the two equations above. One simply needs to substitute out $\theta'(s, 3/2)$ in the second equation using the first one, which gives a relationship between $\theta'(s, 1)$ and $\theta'(s, 2)$ that is satisfied by an infinity of pairs of probabilities.

The fixed point equation for the new state is:

$$B\left(\phi_{3/2}^*, \sum_{s=1,3/2,2,\ldots,n} \theta'(3/2, s) F(\phi_{3/2}^*, \phi_s^*)\right) = 0$$
(A.2)

Let us assume that the probabilities of transition from state 3/2 are weighted averages of the probabilities of transition from states 1 and 2, i.e. $\forall s = 1, 3/2, 2, \ldots, n, \theta'(3/2, s) = \lambda \theta'(1, s) + (1 - \lambda)\theta(2, s)$, where λ is a parameter between 0

and 1. If the transition probabilities from state 3/2 were the same as in state 1, i.e. if λ was equal to 1, then the l.h.s. of Eq. (A.2) would be positive (this results from $\phi_{3/2}^* > \phi_1^*$, $F(\phi_{3/2}^*$, $\phi_s^*) \leq F(\phi_1^*$, ϕ_s^*) and the fixed point equation for state 1). Similarly, one can show that if the transition probabilities were the same as in state 2, the l.h.s. would be negative. This implies, by continuity, that there is one λ between 0 and 1 for which Eq. (A.2) is satisfied. **Q.E.D.**

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