<u>Measurement Uncertainty and Significant Figures</u>

There is no such thing as a *perfect* measurement. Even doing something as simple as measuring the length of a pencil with a ruler is subject to limitations that can affect how close your measurement is to its true value. For example, you need to consider the clarity and accuracy of the scale on the ruler. What is the smallest subdivision of the ruler scale? How thick are the black lines that indicate centimeters and millimeters? Such ideas are important in understanding the limitations of use of a ruler, just as with any measuring device.

The degree to which a measured quantity compares to the true value of the measurement describes the ***accuracy*** of the measurement. Most measuring instruments you will use in physics lab are quite accurate when used properly. However, even when an instrument is used properly, it is quite normal for different people to get slightly different values when measuring the same quantity. When using a ruler, perception of when an object is best lined up against the ruler scale may vary from person to person. Sometimes a measurement must be taken under less than ideal conditions, such as at an awkward angle or against a rough surface. As a result, if the measurement is repeated by different people (or even by the same person) the measured value can vary slightly. The degree to which repeated measurements of the same quantity differ describes the ***precision*** of the measurement.

Because of limitations both in the accuracy and precision of measurements, you can never expect to be able to make an exact measurement. Because of this, in physics lab you are going to record measurements as a range of values within which you are pretty confident the exact value lies. Values will be recorded as

$$\text{best value} \pm \text{uncertainty}$$

where *best value* is your best estimate of the exact value and *uncertainty* is the maximum amount by which you think your measured value might differ from the exact value. Most of the time, you will be taking multiple measurements of the same quantity. In this case, the “best value” of the measurement is the average of the individual measurements. We will use Worst Case methods to determine the uncertainty in a measured quantity as described below.

I. Uncertainty and Accuracy

When performing even a single measurement, your uncertainty should reflect the overall quality of the measurement you take. It is affected both by the resolution of the measuring instrument’s scale and the conditions under which you are performing the measurement. Instruments such as pan balances are designed to measure mass to a certain decimal place with a given uncertainty. The pan balances in our lab are designed to be read to the nearest 0.01 grams ± 0.01 gram, so the uncertainty in a single measurement is 0.01 gram. Micrometer calipers, like pan balances, are designed to provide very accurate measurements to the nearest 1 micrometer (=1 x 10^{-6} m) with an uncertainty of 2 micrometers).

Other measuring tools, such as wooden metersticks or tape measures, are not designed to provide measurements as accurate as those from a micrometer caliper. In physics 107 labs, you will use tools like metersticks to provide good estimates of lengths of about half a meter or greater. Just looking at a meterstick helps to understand its limitations: the black lines indicating the scale on a meterstick are thicker than on either a caliper or even a good ruler, and the wood of the meterstick itself is thick may be slightly warped due to environmental influences, making alignment with the object you are measuring more difficult than with a ruler. Because of these concerns, even under the best conditions you might want to think twice when trying to use a meterstick to measure lengths of even 10 cm or less. Establishing uncertainty in a measurement when using a meterstick is somewhat subjective, but the uncertainty should never be less than 1 mm, and depending on the measurement conditions uncertainty of several mm to 1 cm is possible.

When deciding on uncertainty with a device like a meterstick, particularly if the measurement must be made under non-ideal conditions, some common sense thinking is the best way to proceed. The uncertainty you decide on needs to be a reasonable estimate of the amount your measurement might differ from the ideal value. With a meterstick, factors to consider include not only the characteristics of the meterstick, but also the characteristics of the object you are measuring. For example, if you are measuring the distance between two points marked on paper, are the points really one dimensional or do they have a measurable width? Do you measure the dots center-to-center or from the inside edges of the points? Are the points blurry so that the edge of the point is difficult to establish? When measuring a block of wood, are the edges straight, or are square edges truly square? Such considerations play an essential part in determining uncertainty.

As an illustration of the limitation in use of a meterstick, let's say the meterstick has an uncertainty of 2 mm for a given set of measurements. For a measurement of 0.5 m, 2 mm is less than 0.5% of the measurement, so such a measurement can be considered pretty accurate. However for a measurement of 2 cm measurement, the 2 mm uncertainty is 10% of the measurement. This is substantial uncertainty, and should make you think that another type of measuring device would be more accurate.

For our purposes in lab, all uncertainties should contain only one significant figure (sigfig), such as ± 0.00<u>1</u> m, ± <u>1</u>0 m/s or ± 0.<u>5</u> s. This is the safest way of reporting uncertainty when the measurements you perform do not include substantial in-depth error analysis. A quick example will show the reasoning behind this rule: if you were to indicate an uncertainty of ± 0.15 m, you are saying you are uncertain of your best value in the tenths place (the 0.1), but you are then indicating you know something about the hundredths place. If you are uncertain about the tenths place, how can you know anything about the hundredths place? If, for some reason, you should obtain an uncertainty with more than one sigfig, round the uncertainty to one significant figure: in the above example the ± 0.15 m should round to ± 0.2 m (not ± 0.20 m, because you are indicating with the last zero that you know something about the hundredths place again!!)

When an uncertainty is established for a given measurement, as discussed above, the measurement is written as best value ± uncertainty. As an example, I measure the width of a lab bench and obtain 0.67 m. The measurement is reasonably easy to do so I decide that the uncertainty is ± 2 mm (or 0.002 m). I need to be careful about how I write the value; if I were to write 0.67 m ± 0.002 m I am sending a mixed message about the measurement's accuuracy: the

0.67 m best value says I can measure the length to the nearest 0.01 m and not to any further decimal places. The uncertainty, however, indicates that the value should be accurate to somewhere in the 0.002 m decimal place. So what do you do? **When you decide on the uncertainty, you must report the measurement value to the same decimal place as the uncertainty.** Therefore, if the uncertainty in this measurement is 0.002 m, the best value should be reported to 0.67# m, where # is your best estimate of the digit in the decimal place containing uncertainty. This illustrates the necessity of using a measurement device to its fullest capacity: make sure to record enough digits in a measurement so that its last digit agrees with the uncertainty.

II. Uncertainty and Precision

In physics 107 labs we will often establish uncertainty by performing several measurements of the same quantity. We suggest that each group member performs an independent measurement. It is important when performing independent measurements that group members <u>**not**</u> share measurement information until all group members have performed the measurement (we don't want to hear "I got 2.7 cm – now see what you get" . . . "yeah, I get 2.7 cm too" . . .). While it's human nature that group members want to agree on all measurements, it is only when group members obtain independent (often differing) measurements that the precision of the measurement can be determined. Further, determining the uncertainty of a measured value by comparing multiple trials often brings to light any problems you may not have noticed when performing a single measurement ("Why is your measurement 4 mm longer than mine . . . oh, did you measure to the edge or the center of the dot . . ."). This is often evident when measuring time using digital stopwatches. If the watch reads to 0.001 s, you might decide that an uncertainty of ± 0.001 s is valid for a single-measurement uncertainty. However, if three people perform the measurement simultaneously, their results might differ by a quarter of a second or more – hardly within your ± 0.001 s limit. This forces you to think about the many reasons for the difference: how well the stopwatches are calibrated, how much of a factor is human reaction time, and how clearly evident are the points at which the timing should start and stop are a few possible ideas. Therefore, we will establish uncertainty obtained from multiple measurements or trials whenever possible.

When calculating uncertainty using measurements from multiple measurements or trials, we do so by averaging the measured values to obtain the best value, then using half of the difference between the maximum and minimum measured values to obtain the uncertainty. For example, in your lab group, you perform three independent trials of the same length measurement and record them as 2.7 cm, 2.9 cm and 3.0 cm. The best value will be the average of the three individual values, or 2.867 cm (we'll worry about rounding later) and the uncertainty will be (3.0 cm – 2.7 cm)/2 = 0.15 cm. If you want to record the final result as 2.867 cm ± 0.15 cm you need to think again; above we indicated that the uncertainty must contain only one significant figure and the best value must be rounded to the same decimal place as the uncertainty. For our measurement, then, we must round the uncertainty from 0.15 cm to 0.2 cm and the best value must then be rounded to the tenths place. The final result, then, should be 2.9 cm ± 0.2 cm.

In most cases in physics 107 labs, the uncertainty obtained from multiple measurements will be larger than the uncertainty established from a single measurement; in this case the multiple measurement uncertainty will be used as the overall uncertainty. However, in that event that the uncertainty of the single measurement is larger (for example, in the rare event that group members independently obtain the same measured value), this single measurement uncertainty should be used.

III. Significant Figures

Use of proper rules for significant figures (sigfigs) and mathematical operations with sigfigs can be time consuming AND are of limited benefit for us in what we are trying to learn in physics 107 labs. That said, we need to recognize that while we're not going to be TOO picky about exact rules for sigfigs, we aren't going to abuse them either. For example, I walk a distance of 4.58 m in 7.0 seconds. I calculate my average speed by dividing distance by time: 4.58 m/7.0 s = ??. My calculator tells me the result of this operation is 0.654285714286. If I write this value as my final result (in m/s) I am indicating that I am certain of my speed to 1 thousand billionth of a meter per second. This constitutes sigfig abuse. The distance 4.58 contains 3 sigfigs and 7.0 s contains two. While rules of sigfigs would specify the final result should contain two sig figs, we are only concerned that you recognize the result should be to a few sig figs – not one sigfig and not however many your calculator gives you. Expressing this value as 0.65 m/s or 0.654 m/s is fine; 0.7 m/s is pretty minimal and 0.654285714286 m/s is totally unacceptable. The final result you give should show you are aware of the fact that it was produced using measurements that have a reasonable limit to their accuracy.

IV. Proper Reporting of Units

Note that there are several acceptable methods of reporting quantities with uncertainty, including units. For example, when reporting a length with best value 5.3 cm and uncertainty 1 mm (= 0.1 cm), writing 5.3 ± 0.1 cm would be incorrect because there no units are provided for the best value 5.3. Acceptable methods would be

5.3 cm ± 0.1 cm
(5.3 ± 0.1) cm

or even

5.3 cm ± 1 mm

though with the third method you must be doubly careful to check that the digit to which the best value is rounded corresponds properly with the uncertainty.