b) Since you slip down the slope, use Newton’s second law along the slope to find the component of your acceleration, recognizing that the frictional force on you now is the kinetic frictional force, not the static frictional force, and that your acceleration is no longer zero.

\[ F_{x\text{total}} = ma_x \implies mg \sin \theta - f_k = ma_x \implies mg \sin \theta - \mu_k N = ma_x. \]

The analysis along the y direction is the same as before, so we still have \( N = mg \cos \theta \). Substituting this expression for \( N \) into the last equation we have

\[ mg \sin \theta - \mu_k mg \cos \theta = ma_x \implies a_x = g \sin \theta - \mu_k g \cos \theta = (9.81 \text{ m/s}^2) \sin 30^\circ - (0.45)(9.81 \text{ m/s}^2) \cos 30^\circ = 1.1 \text{ m/s}^2. \]

The magnitude of your acceleration is 1.1 m/s² and you will slide all the way down the slope, going faster and faster.

5.63

a) First convert the speed from km/h to m/s.

\[ v = 200 \text{ km/h} = (200 \text{ km/h}) \left( \frac{10^3 \text{ m}}{\text{km}} \right) \left( \frac{\text{h}}{3600 \text{ s}} \right) = 55.6 \text{ m/s}. \]

Consider a car of the train as the system. The forces on the car as it makes the turn are:

1. the weight \( \vec{w} \) of the car, directed downward; and

2. the normal force \( \vec{N} \) of the track on the car, directed perpendicular to the top surface of the rail.

Minimum wear results when there are no frictional forces either up or down the slope. Introduce a coordinate system with \( \hat{i} \) pointing toward the center of the circular turn (since the centripetal acceleration is along this direction and \( \hat{j} \) pointing straight up. Let \( \theta \) be the banking angle of the track, so \( \theta \) is the angle between \( \vec{N} \) and \( \hat{j} \). Write Newton’s second law for each direction. Recognize that there is no acceleration in the vertical direction, so the total force in this direction is zero, while the acceleration in the horizontal direction is the centripetal acceleration.

\[ \begin{align*}
\text{x direction} & \quad F_{x\text{total}} = ma_x \implies N \sin \theta = m \frac{v^2}{r} \\
\text{y direction} & \quad F_{y\text{total}} = m(0 \text{ m/s}^2) \implies N \cos \theta - mg = 0 \implies N \cos \theta = mg
\end{align*} \]

Divide the last equation on the left by the last on the right to eliminate \( N \).

\[ \tan \theta = \frac{N \sin \theta}{N \cos \theta} = \frac{m \frac{v^2}{r}}{mg} = \frac{v^2}{rg} = \frac{(55.6 \text{ m/s})^2}{(3000 \text{ m})(9.81 \text{ m/s}^2)} = 0.105 \implies \theta = 5.99^\circ \]

b) The rail on the inside of the turn will experience greater wear since the flange of the wheels on this side of the train will be in contact with the inside edge of this rail.

5.64

a) First convert the speed from km/h to m/s

\[ v = 35 \text{ km/h} = (35 \text{ km/h}) \left( \frac{10^3 \text{ m}}{\text{km}} \right) \left( \frac{\text{h}}{3600 \text{ s}} \right) = 9.7 \text{ m/s} \]

The magnitude of the centripetal acceleration of the car is

\[ a_{\text{centripetal}} = \frac{v^2}{r} = \frac{(9.7 \text{ m/s})^2}{150 \text{ m}} = 0.63 \text{ m/s}^2. \]
b) Since the car travels around the curve at constant speed, the tangential acceleration of the car is 0 m/s$^2$.

c) The force providing the centripetal acceleration of the car is found from Newton’s second law. Use the magnitudes of the vectors.

$$F_{\text{total}} = ma = (1250 \text{ kg})(0.63 \text{ m/s}^2) = 7.9 \times 10^2 \text{ N}.$$ 

This force is produced by the static force of friction between the tires and the road.

d) The forces acting on the car are:

1. its weight $\vec{w}$, directed downward;
2. the normal force $\vec{N}$ of the surface on the car, directed up; and
3. the static force of friction $\vec{f}_s$ between the tires and the roadway, directed horizontally towards the center of the turn (in the same direction as the centripetal acceleration).

Let $\hat{i}$ point toward the center of the turn, and $\hat{j}$ point up. There is no acceleration along the $\hat{j}$ direction, so the total force component in this direction is 0 N. Thus

$$F_y \text{ total} = 0 \text{ N} \implies N - mg = 0 \text{ N} \implies N = mg.$$ 

The maximum magnitude $f_{s \text{ max}}$ of the static force of friction is related to the magnitude of the normal force by

$$f_{s \text{ max}} = \mu_s N = \mu_s mg.$$ 

The static force of friction provides the centripetal acceleration, so

$$7.9 \times 10^2 \text{ N} = \mu_s mg = \mu_s (1250 \text{ kg})(9.81 \text{ m/s}^2) \implies \mu_s = 0.064.$$ 

5.65

a) If the hanging mass is zero, the force of the cord on you is zero. In this case, the forces acting on you are:

1. your weight $\vec{w}$, acting downward;
2. the normal force $\vec{N}$ of the surface on you, directed perpendicular to the surface; and
3. a force of friction $\vec{f}$, directed parallel to the surface. (If you do not slip, this is a static force of friction.)

You will slip down the plane if the component of your weight down the plane exceeds the maximum magnitude of the static force of friction up the plane. The component of your weight down the plane is

$$m'g \sin \theta = (70 \text{ kg})(9.81 \text{ m/s}^2) \sin 40^\circ = 4.4 \times 10^2 \text{ N}.$$ 

There is zero acceleration of the system perpendicular to the plane. Choosing a coordinate system with $\hat{j}$ in this direction we have

$$F_y \text{ total} = m'a_y \implies N - m'g \cos \theta = m'(0 \text{ m/s}^2) \implies N = m'g \cos \theta.$$ 

The maximum magnitude of the static force of friction is

$$f_{s \text{ max}} = \mu_s N = \mu_s m'g \cos \theta = 0.40(70 \text{ kg})(9.81 \text{ m/s}^2) \cos 40^\circ = 2.1 \times 10^2 \text{ N}.$$ 

Since the component of your weight down the incline is greater than the maximum magnitude of the static force of friction, you will slide down the plane.