b) Let \( \vec{F} \) be the force you are exerting to hold the mass in position. The forces acting on the mass \( m \) are:

1. the force \( \vec{F} \) that you exert;
2. the force of spring 2 on \( m \), which equals \( -k_2(x - x')\hat{i} \), since \( (x - x') \) is the extension of the spring and the force is acting in the \(-\hat{i}\) direction.

Since the mass is not accelerating, the total force on it is zero. Therefore, using part a)

\[
\vec{F} - k_2(x - x')\hat{i} = 0 \text{ N} \implies \vec{F} - k_2x\left(1 - \frac{k_2}{k_1 + k_2}\right)\hat{i} = 0 \text{ N} \implies \vec{F} - \frac{k_1k_2}{k_1 + k_2}x\hat{i} = 0 \text{ N}.
\]

If you replace the dual springs with a single spring, with force constant \( k_e \), that is stretched and held at a distance \( x \), then

\[
\vec{F} - k_e x\hat{i} = 0 \text{ N}.
\]

Compare this with the previous equation and find

\[
k_e = \frac{k_1k_2}{k_1 + k_2}.
\]

This result is sometimes stated as “spring constants in series have reciprocals that add,” since

\[
k_e = \frac{k_1k_2}{k_1 + k_2} \iff \frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}.
\]

7.10 The combined spring has force constant \( k \), which is called \( k_e \) in Problem 7.9 part b). Let \( k' \) be the spring constant of each half of the original spring. Then, since the original spring is cut in half, \( k_1 = k_2 = k' \) in Problem 7.9 part b), and therefore

\[
k_e = \frac{k_1k_2}{k_1 + k_2} \implies k = \frac{k'^2}{2k'} \implies k' = 2k.
\]

7.11 Let \( \hat{i} \) be in the direction corresponding to stretching the spring. Then

\[-kx = ma_x.\]

Since the spring is compressed \( 0.050 \text{ m} \), then \( x \) is negative \( (x = -0.050 \text{ m}) \) therefore

\[-(30 \text{ N/m})(-0.050 \text{ m}) = (0.100 \text{ kg})a_x \implies a_x = 15 \text{ m/s}^2.\]

So the magnitude of the acceleration is \( 15 \text{ m/s}^2 \).

7.12

a) The amplitude of the oscillation is the coefficient of the cosine term, \( A = 6.0 \text{ m} \).

b) The angular frequency is the coefficient of \( t \) in the cosine term, \( \omega = 8.00 \text{ rad/s} \).

c) The frequency is

\[
\nu = \frac{\omega}{2\pi} = \frac{8.00 \text{ rad/s}}{2\pi} = 1.27 \text{ Hz}.
\]

d) The period is

\[
T = \frac{1}{\nu} = \frac{1}{1.27 \text{ Hz}} = 0.787 \text{ s}.
\]

e) The effective spring constant is

\[
k = m\omega^2 = (50.0 \text{ kg})(8.00 \text{ rad/s})^2 = 3.20 \times 10^3 \text{ N/m}.
\]