6.19  Use Kepler’s third law of planetary motion:

\[ T^2 = \frac{4\pi^2 r^3}{GM} \]

with M as the mass of the Earth and r as the radius of the orbit of the satellite. Here \( r \) is 200 km greater than the radius of the Earth, so

\[ r = 6.37 \times 10^6 \text{ m} + 200 \times 10^3 \text{ m} = 6.57 \times 10^6 \text{ m}. \]

Hence

\[ T = \sqrt{\frac{4\pi^2 (6.57 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}} = 5.30 \times 10^3 \text{ s} = 88.3 \text{ min}. \]

6.20  

a) Write Kepler’s third law of planetary motion for each satellite:

\[ T_1^2 = \frac{4\pi^2 r_1^3}{GM} \quad \text{and} \quad T_2^2 = \frac{4\pi^2 r_2^3}{GM}. \]

Divide the first equation by the second, simplify, and take square roots

\[ \frac{T_1^2}{T_2^2} = \frac{4\pi^2 r_1^3}{4\pi^2 r_2^3} \quad \Rightarrow \quad \frac{T_1}{T_2} = \sqrt{\frac{r_1^3}{r_2^3}}. \]

b) If \( r_2 = 2r_1 \) then

\[ \frac{T_1}{T_2} = \sqrt{\left(\frac{1}{2}\right)^3} = \sqrt{\frac{1}{8}} = 0.354, \]

so \( T_2 = \frac{1}{0.354} T_1 = 2.82 T_1 \).

6.21  Consider the mass \( m \) in free-fall near the surface of Mars. Let \( R \) be the radius of Mars and \( M_{Mars} \) its mass. The only force on \( m \) is the gravitational force of Mars on it. Therefore

\[ F_{\text{total}} = ma \Rightarrow \frac{GM_{Mars}m}{R^2} = ma \Rightarrow M_{Mars} = \frac{aR^2}{G} = \frac{(3.776 \text{ m/s}^2)(3.37 \times 10^6 \text{ m})^2}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2} = 6.43 \times 10^{23} \text{ kg}. \]

Then the ratio

\[ \frac{M_{Mars}}{M_{Earth}} = \frac{6.43 \times 10^{23} \text{ kg}}{5.98 \times 10^{24} \text{ kg}} = 0.108. \]

The mass of Mars is only about 11% the mass of the Earth.

6.22  

a) Use Kepler’s third law of planetary motion.

\[ T^2 = \frac{4\pi^2 r^3}{GM} \]

where M is the mass of the Earth and r is the radius of the orbit of the satellite. Solve for the radius.

\[ r = \sqrt{\frac{GMT^2}{4\pi^2}}. \]

The period \( T \) of the satellite is one day = 8.6400 \times 10^4 \text{ s}. Therefore

\[ r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(8.6400 \times 10^4 \text{ s})^2}{4\pi^2}} = 4.23 \times 10^7 \text{ m}. \]
b) The speed of the satellite is
\[ v = \frac{2\pi r}{T} = \frac{2\pi (4.23 \times 10^7 \text{ m})}{8.6400 \times 10^4 \text{ m}} = 3.08 \times 10^3 \text{ m/s}. \]

c) The acceleration of the satellite is
\[ a_{\text{centripetal}} = \frac{v^2}{r} = \frac{(3.08 \times 10^3 \text{ m/s})^2}{4.23 \times 10^7 \text{ m}} = 0.224 \text{ m/s}^2. \]

6.23

a) Here’s a view looking down from above the north pole.

Both satellites orbit the Earth (of mass \( M \)) with the same orbital radius. According to Kepler’s third law of planetary motion
\[ T^2 = \frac{4\pi^2 r^3}{GM} \]
this implies that both will have the same orbital period.

b) They will pass each other twice during each orbital period, at the locations labeled A and B in the sketch above.

c) Since the Earth is rotating in the sense indicated in the sketch above, completing one rotation every 24 h, the places where the satellites pass each other will not, in general, remain over the same locations on the surface of the Earth. (But what if the east-moving satellite is a communications satellite, so that it is always above the same point on Earth?)

d) Since the Earth rotates to the east, the satellite moving west will have the shorter time interval between successive passages over the same place on the equator of the Earth.

6.24 Refer to Figure P.24 on page 273 of the text

a) The hypotenuse of the right triangle is \( R + h \), so
\[ \frac{R + h}{R} = \sec \theta \implies h = R \sec \theta - R = R(1 - \sec \theta). \]

b) The secant is the reciprocal of the cosine, so
\[ \sec \theta = \frac{1}{\cos \theta} \approx \frac{1}{1 - \frac{\theta^2}{2}} = \left(1 - \frac{\theta^2}{2}\right)^{-1}. \]

Use the binomial expansion. For small values of \( x \)
\[ (1 + x)^n \approx 1 + nx. \]