Lab: Acceleration and Force: II

Name ___________________________ Partners ___________________________

Pre-Lab

You are required to finish this section before coming to the lab, which will be checked by one of the lab instructors when the lab begins.

When a force is applied to any object, a small amount of deformation can occur. For example, when masses are hung from the bottom of a spring, the spring will deform, or stretch, according to the total amount of mass present. Robert Hooke studied this topic and realized that the stretch of the spring is proportional to the force exerted by the hanging mass. This concept is referred to as Hooke’s Law and states that

\[ F = k \Delta L \]

in which \( F \) is the force, \( k \) is the proportionality constant that depends on the shape and composition of the spring and \( \Delta L \) is the distance the spring stretches. \( k \) is commonly called the “spring constant”. Hooke’s Law assumes that you are not adding so much mass that you stretch the spring beyond its elastic limit (where it becomes a wire).

While Hooke investigated spring stretch using the force generated by hanging masses on the spring, a force generated by any method will cause the spring to react in the same way. In this lab, you are going to provide force to the spring using two different methods, then compare the amount of force needed in each method to stretch the spring by the same specified amount. The second method will parallel what Robert Hooke did: you will hang mass on the spring and try to produce the specified amount of stretch by adding just the right amount of mass to the spring. In the first method, you are going to attach a known mass to the spring and rotate the mass/spring system. The amount of spring stretch is proportional to the rotation speed of the system, so you can achieve the specified amount of spring stretch by finding the ideal rotation speed of the system.

Read your textbook and class notes on circular motion. Consider an object moving along a circle at a uniform speed. If you measure the total time, \( T_N \), the object takes for \( N \) revolutions and the radius of the circle, \( R \). How do you find the magnitude of the centripetal acceleration from your data? Derive a formula giving \( a_c \) in terms of \( R \), \( N \), and \( T_N \). Hints: What is the distance the object moves as it completes one revolution around the circle? How long will it take for the object to complete one revolution if \( N \) revolutions takes time \( T_N \)?
Read your textbook and class notes on Newton’s three laws of motion. Write down the formulas that represent Newton’s second and third laws.

1) A block of 0.10 kg is attached to one end of a spring. The other end of the spring is attached to the ceiling. The block is at rest.

   i) Sketch your visualization of the given case.  
   ii) Draw the free-body diagram for the block. Label each force vector and describe each force briefly.

2) Draw a tip-to-tail diagram summing the forces on the block in the x and y directions.

3   a) Determine an equation for the force on the block by the spring \( F_{bs} \) and calculate that force.

   b) Determine the magnitude of the force on the spring by the block \( F_{sb} \) and state your reasoning.

- end of prelab -
I. Spring Force

1) Consider the same spring used in prelab question 1. This time one end of the spring is fastened to the wall. You pull the other end to make it stretch by the same amount as in prelab question 1.

\[ F_{sy} \]

a) The vector in the above figure represents the force on the spring by you (\( F_{sy} \)). Draw the vector representing the third law force pair for the above force and label it correctly.

b) Determine the magnitude of the spring force and clearly state your reasoning.

II. Rotating Cage

Before you start any measurements, let’s review the idea of uncertainty. No measurement is perfect. The accuracy of a measurement is limited by the measurement instrument itself, by the observer’s limitation in reading it correctly, etc. We calculate uncertainty to account for these limits in our measurements. In the following experiments, think of the accuracy of the instrument when you make a measurement. Also, each member in your group should take a trial. You will see that the value of the measurement differs slightly for each individual even though everyone takes the same measurement.

The main apparatus includes a centripetal force cage and motorized rotator. Examine the centripetal force cage, comparing it to the figure below. It clamps to a shaft on top of the rotator. When the cage rotates, the spring stretches from its initial length by a certain amount, depending on the velocity of the rotation.

Centripetal Force Cage
1) Why does the spring stretch? (Hint: Think of Newton’s third law.)

When the cylinder in the cage is at a certain critical distance from the rotation axis, it touches the pointer arm, lifting the pointer into alignment with the index. An adjustment screw attached to the spring permits the initial length of the spring to be varied. This is preset for you and should not be adjusted.

Make certain the cage is securely fastened to the rotator. Turn the speed adjusting screw at the bottom of the rotation mechanism until the rubber wheel is about 1 cm from the center of the brass disk. This ensures that the speed of the cage will be low when the motor is turned on. The spring tension adjustment should already be set at about 10 mm.

Start your rotator. **Warning:** Keep important body parts, hair, sleeves etc. away from the spinning cage. Increase the rotation rate until the cylinder touches the pointer arm. Practice to keep the rotating rate near the “critical value” (the rate required to keep the tip of the pointer centered on the index). **Note:** the pointer does not extend very far beyond the index, so be careful that you are not “pegging” the pointer at the top of its range by rotating the cage too fast. To make sure this doesn’t happen, check to make sure that reducing the rotation rate slightly will cause the pointer to fall. You may find that you will need to adjust the rotation rate continuously during your measurement periods. Do not worry about the pointer rising and dropping during measurement; as long as you maintain the system near the critical value, slight fluctuations in the pointer will have minimal effects on your data. If you have difficulty seeing the pointer when the cage is rotating, try holding a piece of white paper behind the cage and focus on the index. Then try to find the pointer, which should appear to point towards the index axis from both sides.

For each reading one person in your group should maintain the rate, another person should operate the revolution counter, and the last should operate the timer.

2) Measure the number of revolutions, made in a total time, $T_N$, between 50 to 100 seconds. Make certain to record the starting value of the counter and the actual time taken for the number of revolutions you counted. Switch roles in the group so that everyone takes a trial. Organize your data in a table. **NOTE:** It may be necessary to practice this a few times so you are all consistent. If there are data points that do not fit the trend of the rest of the results then take the data point again. It is important to minimize uncertainty.
3) Calculate the period, T, (time for 1 revolution) for each trial.

4) Find the average period of the trials. This average period is your best result for the period.

5) Use worse case analysis to find the uncertainty of your period.
   
   Reminder: your uncertainty should include only one significant figure.

6) State your final result of period as best result ± uncertainty, making sure to include units.

7) Assuming the critical rotation radius is (5.5 +/- 0.1) cm, use the expression you derived in pre-lab and the best results of the measured data to calculate your best value for the magnitude of the acceleration of the cage. You do not need to perform uncertainty calculations here; the uncertainty in the rotation radius given above is for use in the homework section.

8 a) Record the value stamped on the cylinder, record the mass of the cylinder, m₀, and its uncertainty. Use the following example to determine uncertainty in the mass:

   Example: stamped mass = 324.75 g
   
   Step 1: Write the value with 0 as an additional significant digit: 324.750 g
   
   Step 2: Find the max and min values for masses that will round to your stamped mass:
   
   min value = 324.745 g (since 324.744 g would round to 324.74 g)
   
   max value = 324.754 g (Since 324.755 g would round to 324.76 g)
   
   Step 3: Perform worst case analysis on max and min values to determine uncertainty.
   
   uncertainty = (324.754 g – 324.745 g)/2 = 0.0045 g or 0.005 g to 1 sig fig.
   
   The mass value with uncertainty is 324.750 g ± 0.005 g.

b) Use Newton’s second law to find the best value for the magnitude of the spring force (again, no uncertainty calculations are needed right now). Include appropriate SI units.
III. Hanging Mass

We are now going to use the second method discussed in the prelab to stretch the spring until the pointer arm points to the index. Remove the cage from the rotator and hang it from the ring stand. If you hang “extra” mass (mass in addition to the cylinder mass $m_o$) from the cylinder then the spring will stretch.

1) Using your results from part II, question 8, predict how much “extra” mass would you need to hang from the spring to stretch it until the pointer just aligns with the index. State your reasoning.

2) Now you will check your prediction by hanging weights from the cylinder to determine the “extra” mass, in addition to $m_o$, needed to stretch the spring just enough to align the pointer with the index. First, each group member should use a balance independently to find the mass of the hanger to the nearest 0.01 g. If the mass of your hanger seems to exceed the capability of the scale, there are additional masses that can be hung from the balance to enable measurement of greater masses. Ask a lab instructor about their use if you are unfamiliar with them. Write down your measurement but do not reveal your values to other members in your group until the end.

3) Now, as a group, add additional mass to the hanger until the spring is stretched to exactly the same length as it was in the rotating cage method. For uncertainty purposes, the hanging masses should be considered to have a tolerance of 2% (the actual mass can differ from the ideal mass by up to 2%). Determine and record the total mass hanging from the spring and its uncertainty. (Remember to include all of the mass hanging from the spring.)

4) Calculate the best value for magnitude of the gravitational force using the average total mass you have found. Is it close to your prediction in question 1?

Check with an instructor before you leave.
Homework Problems

1. a) Draw free-body diagrams of the cylinder in the cage when (i) it rotates at the critical rate, and (ii) when extra weights were hanging from it. In each diagram, label each force and write a brief description next to the vector.

b) For each free-body diagram, draw a tip-to-tail diagram summing the forces in each direction. Make sure all forces are clearly labeled on the diagram.

c) On either of the free-body diagrams above, is there a net force in any direction? If so, what is it and which direction it is pointing?
Data Analysis for Force caused by Rotating Cage

2 a) For the rotating cage, write down your results for the period (time for one revolution), radius, and mass of the cylinder. Each result should be given in the form of average ± uncertainty. Include appropriate SI units.

b) Calculate the best result for the force.

c) Use worse case analysis to find the uncertainty in force. (To find $F_{\text{max}}$, use the maximum value of the measured quantities in numerator and the minimum value of the measured quantities in denominator. For $F_{\text{min}}$, use the minimum value of the measured quantities in numerator and the maximum value of the measured quantities in denominator.)

d) Write your final result as (best +/- uncertainty) and be sure to include the proper SI units.
Data Analysis for Force caused by Hanging Mass

3  a) For the hanging mass, write down your results for the mass of the cylinder, extra mass, and the
mass of the hanger. Each result should be given in the form of average ± uncertainty. Include
appropriate SI units.

b) Calculate the ‘best result’ of the gravitational force required to stretch the spring so the pointer
aligns with the index.

c) Find the uncertainty in force using worst case methods to maximize and minimize the mass
attached to the spring.

d) Write your final result as (best +/- uncertainty) and be sure to include the proper SI units.

4  a) Compare your results for the forces calculated in each method. Do they agree within
uncertainty limits (do the ranges of allowed values for the forces overlap)? Show the ranges of
each force result on a single number line to illustrate your response.

b) List and describe several physical factors that led to uncertainty in your measurements.