Lab Section (circle):
Day: Monday Tuesday
Time: 8:00 9:30 1:10 2:40

Electron Charge-to-Mass Ratio

Name________________________________________ Partners________________________________________

**Pre-Lab**

*You are required to finish this section before coming to lab; it will be checked by one of the lab instructors when the lab begins.*

The purpose of this lab is to determine the charge-to-mass ratio \( \frac{e}{m} \) of an electron by measuring the radius \( R \) of the circular path it follows when moving with a known velocity in a given magnetic field. Historically this experiment was first performed in 1897 by J. J. Thomson to show that “cathode rays” were particles given off by atoms rather than atoms or ions themselves. In 1909 Millikan measured the charge carried by a “cathode ray.” Taken together, the experiments of Millikan and Thomson identified “cathode rays” as electrons, and provided the first determination of their charge and mass. Millikan and Thomson each won the Nobel Prize for their work.

In our version of the experiment, an electron is accelerated from rest to a speed \( v \) and then sent into a region with a uniform magnetic field. First let’s discuss how the electron is accelerated. As you know, a cathode is a material with an excess of electrons. In our experiment, a metal cathode is heated by a filament until some of the excess electrons have enough energy to escape. The cathode is only hot enough to free the electrons so, once they are free, the electrons are essentially at rest. Across from the cathode is an anode, which is a metal with an excess of positive charge. The electrons that came off the cathode accelerate toward the anode. Since the \( \mathbf{E} \) field between the cathode and anode is uniform, the acceleration is constant and you can use the usual equations of kinematics to calculate the speed of the electron when it reaches the anode. The result is

\[
v = \sqrt{\frac{2eV}{m}},
\]

where \( e \) is the electron charge, \( m \) is the electron mass, and \( V = Ed \) is the accelerating voltage, where \( d \) is the distance between the cathode and the anode (the distance through which the electron travels).

1) Using the force law \( \mathbf{F} = q\mathbf{E} \) in Newton’s second law, show that the acceleration of the electron is \( \mathbf{a} = (-\frac{e}{m})\mathbf{E} \). Put this acceleration into one or two of the kinematics equations and derive Eq. (1).
After passing through the anode the electron enters a magnetic field $\mathbf{B}$ which exerts a force on it of

$$
\mathbf{F} = (-e) \mathbf{v} \times \mathbf{B}.
$$

Since $\mathbf{F} = m\mathbf{a}$, the acceleration of the electron is always perpendicular to its velocity, which means the electron moves in a circular path at constant speed.

For this lab, the magnetic field $\mathbf{B}$ is produced by two rings of wire, called “coils”, which sit in parallel planes. The coils are arranged so that this separation is equal to their radius. Two coils of wire with this geometry are called “Helmholz coils”. The essential property of Helmholz coils is that they produce a uniform $\mathbf{B}$ field at the point midway between them. The magnetic field strength, $\mathbf{B}$, is related to the Helmholz coil current, $I$, as follows:

$$
\mathbf{B} = \frac{8\mu_0 N I}{a\sqrt{125}}
$$

where $\mu_0 = 4\pi \times 10^{-7}$ T m/A is the permeability of space, $N$ is the number of turns in each coil, and $a$ is the radius of each coil.

2) What is the accepted value for $e/m$ in MKS units?

3) In the rotating cage lab, you used a spring to provide the centripetal force required to keep a metal cylinder moving in a circle. In this lab, it is the force in Eq. (2) that provides the centripetal force needed to keep the electron in a circular path. Using $\mathbf{F} = m\mathbf{a}$, put the force law of Eq. (2) in the left hand side and the centripetal acceleration in the right hand side and find an equation for $e/m$ in terms of $V$, $\mathbf{B}$ and $R$. Notice that the $V$ you want to include is voltage $V = Ed$, not the velocity. Your equation for $e/m$ should not include the velocity $\mathbf{v}$, so you will need to use equation (1) to express velocity in terms of the allowed quantities.

4) In Eq. (2) the “x” between the $\mathbf{v}$ and $\mathbf{B}$ refers to the vector cross product.

   a) Using the definition of the cross product, write an equation expressing the magnitude of the force as a function of the angle between $\mathbf{v}$ and $\mathbf{B}$.

   b) An electron moving due north encounters a magnetic field directed due east. What is the direction of the force exerted on the electron? *Hint: Careful! What is the sign of the electron charge?*

To make sure you are prepared for lab, read the **Procedure** section below and feel free to ask questions!

**End of Pre-Lab.**
**Apparatus**

Leybold fine-beam e/m tube  
Helmholz coils  
Illuminated scale with reflecting glass plate  
5 Banana to Banana wires  
2 DC outlet plugs (one spade, one banana)

DC Ammeter (0-3Amps)  
Rheostat (20 Ohms)  
Pasco SF-9585A DC Power supply  
1 Banana to Spade wire

**Procedure**

Examine the apparatus. The Leybold fine-beam tube contains a cone shaped metal anode with a small hole in its apex. This is the hole through which electrons will be ejected with a velocity $v$. Hidden within the conical anode is a cathode and heater filament. Surrounding the tube are the two Helmholz coils. According to the manufacturer, each of the coils has exactly 130 turns of wire.

**WARNING:** The output voltage provided by the power supply can be LETHAL! Don't touch any terminals or wires connected to the supply unless it is unplugged from the AC wall outlet.

For time and safety reasons, the circuit connections between the power supplies and the e/m apparatus have been pre-wired. Examine them carefully, to make sure they are wired as described below. The high-voltage output terminals are on the e/m apparatus are labeled with a '+' and a '-'. These positive and negative terminals should be connected to the terminals on the power supply that are labeled 0 and 500V with banana-to-banana wires. The light on the upper part of the power supply should read 500 Volts. If this is not the case, press the gray button below the indicator until the 500-Volt light is lit.

Study the controls on the high voltage power supply. Make sure the supply is unplugged and the power switch is off. Find the terminals that provide 6 volts AC on the right side of the power supply. Use two banana-to-banana wires to connect them (in any order) to the heater inputs on the e/m apparatus (labeled 6.3V). Check to make sure the dial on the power supply is set on 6 V and leave it there. **Leave the high voltage supply unplugged and turned off for now.**

![Helmholz Coil Setup](image)

Figure 1: Helmholz Coil Setup
The circuit in Figure 1 shows the electrical circuit used to wire the Helmholz coils. This is a low voltage circuit, so you can touch its various parts without fear. The required 6 Volts comes from the DC terminals on the wall, not the high voltage power supply. **Leave one input to the wall power unplugged until you are ready to start the lab.** Make certain the “10A” terminal on the ammeter is connected to the “+” power terminal. The order of connections for the coils is also important, and they may be connected properly or backwards, so when you first see the beam and adjust the current you’ll have to decide whether or not they’re connected properly. You can reverse them yourself if you decide they’re backwards, but be sure to turn the high voltage power supply off before you do so.

![Figure 2: Use of Illuminated Scale](image)

Observe the illuminated scale with its attached reflecting glass plate. When plugged into the AC outlet, the scale lights up, and if the room lights are off you will be able to see a reflected image of the scale by looking into the glass with the scale itself facing away from you, as shown in Figure 2. Note that the distance of this image behind the glass is equal to the distance between the scale and the glass, and that the magnification of this optical system is exactly 1.

Measure the diameter of each coil and the separation between coils. Keep in mind the relationship between the two when calculating uncertainty. Turn on the power supply and let the filament in the fine-beam tube warm up for a few minutes. Then, with the room lights off, turn the anode voltage up to about 150 volts or until the electron beam is seen emerging from the hole in the “electron gun”. Plug in the Helmholz coil circuit and adjust the rheostat until the beam trajectory is a full circle. If the beam curves in the wrong direction, reverse the connections at the coil inputs on the e/m apparatus. Observe the effect of grasping the fine-beam tube and rotating it about the horizontal axis through its mounts. **Turn the tube only, not the mounts or Helmholz coils.**

1) What do you observe when you turn the tube. Why does this occur?

Rotate the tube until the trajectory is circular.

To measure the diameter of the circle in which the electrons are traveling, hold the illuminated scale and reflecting glass plate so the virtual image of the scale is positioned exactly in the plane of the electron path. Remember the distance between the glass plate and the virtual image of the scale: if you position the scale image in the plane of the electron orbit, you can measure the orbit diameter just as if there were a real scale inside the fine-beam tube.

For one setting of current and voltage, measure the beam diameter. Also measure the radius of the Helmholz coils (referred to as “a”). A computer will be set up with an Excel template for you to quickly calculate a value for \( e/m \) using your data. If your result for \( e/m \) is not within 15% of the accepted value, find the source of the discrepancy. Once you are getting a result that is within 15% of the accepted value, measure the diameters of orbits for 10 combinations of anode voltage and coil current, recording in tabular form the values of the three measured quantities.
Place your data table on the following page. For at least one voltage/current combination, be sure to have each lab group member measure the beam diameter independently to establish data for uncertainty in diameter.

**Data Table:**
2) The quantity \( a \) is the radius of each of the Helmholtz coils, and is also the separation distance between the coils. Use your measured radius and separation to establish an uncertainty for \( a \). Explain how you decided on your uncertainty.

**Sample Calculations**

Find a value for \( e/m \) using your first row of data and the formulas for \( e/m \) for \( B \) from the pre-lab section. Show all calculations.

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**Turn off the power supply and unplug the 6 VDC wall supplies when you are finished.**

*Check with an instructor before you leave.*
Homework

I. Data Analysis

Set up an Excel worksheet that uses your pre-lab work and the equation for B in the pre-lab to calculate the best e/m and the max/min e/m for each of your data sets of V, I, and R. A suggestion of how to set up columns in your spreadsheet follows. Note: for this experiment, we are going to neglect uncertainty in voltage and current because it will be negligible when compared to your uncertainty in a and R. Typically, uncertainty from a digital readout such as the current and voltage readouts is performed (assuming all digits are non-fluctuating) by adding a decimal place to the right of the smallest one available on the readout and using a “5” in this digit for uncertainty. For example, a reading of 2.53 A from the meter would be written as 2.530 A ± 0.005 A. In your spreadsheet, use short column headings to make the most use of the space on the page, and don’t forget units (place them in the cell below the heading to save space)!

Column A: best a
Column B: max a (this is probably obvious, but if you enter the cell address of the average value for a from the cell in column A and add your uncertainty in a, you can just copy and paste the equation for all sets of data)
Column C: min a (set this up in a manner similar to column B)
Column D: best R
Column E: max R
Column F: min R
Column G: I
Column H: V
Column I: best e/m
Column J: max e/m
Column K: min e/m

II. Questions

1) Write your result for best value of e/m ± uncertainty in e/m. For the uncertainty in e/m, do worst case analysis using all of the best e/m values from your trials. Show all calculations.

2) Calculate the percent error between your best e/m value and the ideal value you calculated in the pre-lab. What single factor in performing this lab do you think explains this difference? Explain why. Hint: look at the effect on the result of uncertainty in each measured quantity on the result to support your answer.
3) Look at the ranges of uncertainty for each set of data in your spreadsheet and compare these to the uncertainty from question 1. Which uncertainty range is larger? Why do you think this is the case?