A BRIEF OVERVIEW OF PARTIAL UNCERTAINTY

In physics 108 lab we will be analyzing uncertainty in data mostly using *partial uncertainty analysis*. In physics 107 lab, uncertainty analysis was done using *worst-case uncertainty analysis*. A quick example of each should illustrate the difference.

We will use an example of a density calculation to show how to use each analysis type.

In lab, you are given a solid for which you need to determine the density. Using a balance, you find the mass of the solid to be 341.5 g ± 0.1 g. Using a graduated cylinder, you find the volume (by water displacement) to be 101.3 cm³ ± 0.2 cm³.

**Worst-Case Uncertainty Analysis**

To find the density, we need to use the equation

\[
\text{density} = \frac{\text{mass}}{\text{volume}}, \quad \text{or} \quad d = \frac{m}{V}
\]

To find the best (most probable) value of density, just calculate using the measured mass and volume without uncertainty:

\[
d = \frac{341.5\text{g}}{101.3\text{cm}^3} = 3.371 \ \frac{\text{g}}{\text{cm}^3}
\]

To find the uncertainty in density, we need to find the maximum and minimum possible calculated densities based on the measured mass and volume, including measurement uncertainty. As we would have done in physics 107, the max and min densities would be calculated as follows:

\[
d_{\text{max}} = \frac{m_{\text{max}}}{V_{\text{min}}} \quad \text{and} \quad d_{\text{min}} = \frac{m_{\text{min}}}{V_{\text{max}}}
\]

Plugging in:

\[
d_{\text{max}} = \frac{341.6\text{g}}{101.1\text{cm}^3} = 3.379 \ \frac{\text{g}}{\text{cm}^3} \quad \text{and} \quad d_{\text{min}} = \frac{341.4\text{g}}{101.5\text{cm}^3} = 3.364 \ \frac{\text{g}}{\text{cm}^3}.
\]

The uncertainty is half the difference between the values:

\[
\frac{3.378 \ \frac{\text{g}}{\text{cm}^3} - 3.365 \ \frac{\text{g}}{\text{cm}^3}}{2} = 0.008 \ \frac{\text{g}}{\text{cm}^3}.
\]

The overall density value, then, is the best value ± uncertainty, or 3.371 \ \frac{\text{g}}{\text{cm}^3} ± 0.008 \ \frac{\text{g}}{\text{cm}^3}. This could also reasonably be written as 3.37 \ \frac{\text{g}}{\text{cm}^3} ± 0.01 \ \frac{\text{g}}{\text{cm}^3}.
Partial Uncertainty Analysis

Using partial uncertainty analysis, we will use the same density equation (of course), but we’ll deal with the effect of uncertainty in each measurement separately. Partial uncertainty analysis is much easier to do than worst-case uncertainty in more involved calculations. For example, in the equation

\[ n = \frac{(2t - N\lambda)(1 - \cos \theta)}{2t(1 - \cos \theta) - N\lambda} \]

which you will be using this semester(!), to find the maximum value for \( n \) using worst-case analysis it’s not immediately obvious whether to use the maximum or minimum values for measured \( t, N, \Theta, \) and \( \lambda \). To be sure you got the maximum \( n \) you’d need to do a lot of work using worst-case analysis. Using partial uncertainty, you would calculate the effect of plugging in max and min values of each measured value separately – a lot easier in the long run!

Back to our density example: our two measurements are mass and volume. For partial uncertainty analysis, first find the “best” value for density using the measured mass and volume without uncertainty as we did for the worst-case method. Then, look at uncertainty for each measured quantity separately:

**mass**

\[
d\ (using\ m_{\text{max}}) = \frac{341.6g}{101.3cm^3} = 3.372\ \frac{g}{cm^3} \text{ and } \\
d\ (using\ m_{\text{min}}) = \frac{341.4g}{101.3cm^3} = 3.370\ \frac{g}{cm^3}.
\]

Notice since we’re looking at uncertainty only in mass here, we use the best value for the volume. Uncertainty in density due to mass is

\[
\frac{3.372\ \frac{g}{cm^3} - 3.370\ \frac{g}{cm^3}}{2} = 0.001\ \frac{g}{cm^3}.
\]

**volume**

\[
d\ (using\ V_{\text{max}}) = \frac{341.5g}{101.5cm^3} = 3.365\ \frac{g}{cm^3} \text{ and } \\
d\ (using\ V_{\text{min}}) = \frac{341.5g}{101.1cm^3} = 3.378\ \frac{g}{cm^3}.
\]

As above, since we’re looking only at uncertainty in volume here, we use the best value for mass. Uncertainty in density due to volume is

\[
\frac{3.378\ \frac{g}{cm^3} - 3.365\ \frac{g}{cm^3}}{2} = 0.007\ \frac{g}{cm^3}.
\]

The hard part is done: the total uncertainty is the sum of the partials: \( 0.001 \frac{g}{cm^3} + 0.007 \frac{g}{cm^3} = 0.008 \frac{g}{cm^3} \). Our final calculated density, then, is \( 3.371\ \frac{g}{cm^3} \pm 0.008\ \frac{g}{cm^3} \).